

**CC7220-1**

**LA WEB DE DATOS**

**PRIMAVERA 2020**

## **LECTURE 5: WEB ONTOLOGY LANGUAGE (OWL) [II]**

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LAST TIME ...

# SEMANTIC WEB: LOGIC

## DATA:

Ireland



(Ireland,partOf,Europe)  
(Ireland,isA,Country)  
(Ireland,capital,Dublin)

Dublin



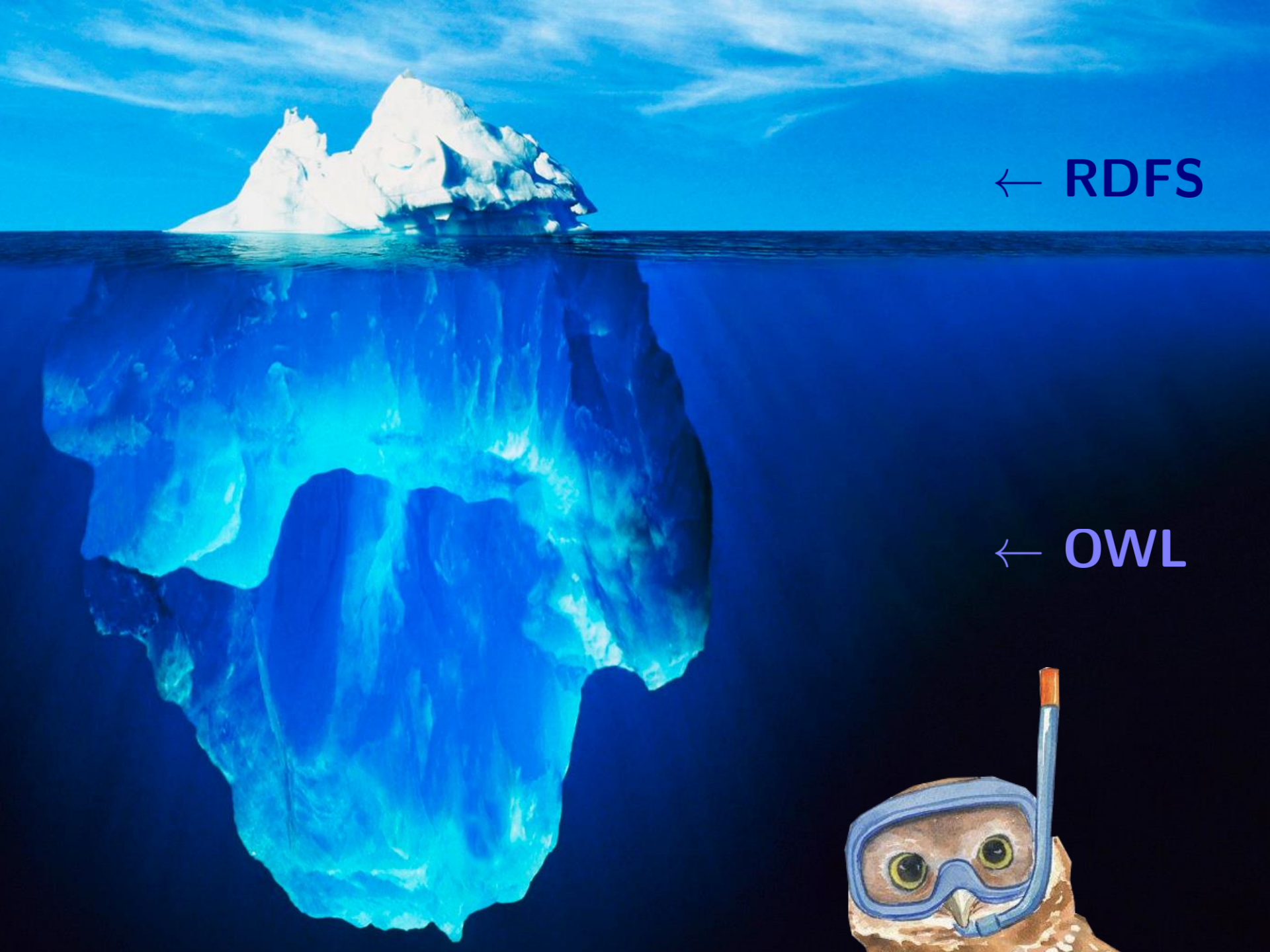
(Ireland,capital,Dublin)  
(Dublin,population,1000000)

LOGIC:  $((b, \text{capital}, a) \rightarrow (a, \text{partOf}, b))$   
 $((a, \text{partOf}, b), (b, \text{partOf}, c) \rightarrow (a, \text{partOf}, c))$

QUERY:  $((x, \text{partOf}, y)?)$

OUTPUT:  $\{(x \mapsto \text{Ireland}, y \mapsto \text{Europe}),$   
 $(x \mapsto \text{Dublin}, y \mapsto \text{Ireland}),$   
 $(x \mapsto \text{Dublin}, y \mapsto \text{Europe})\}$



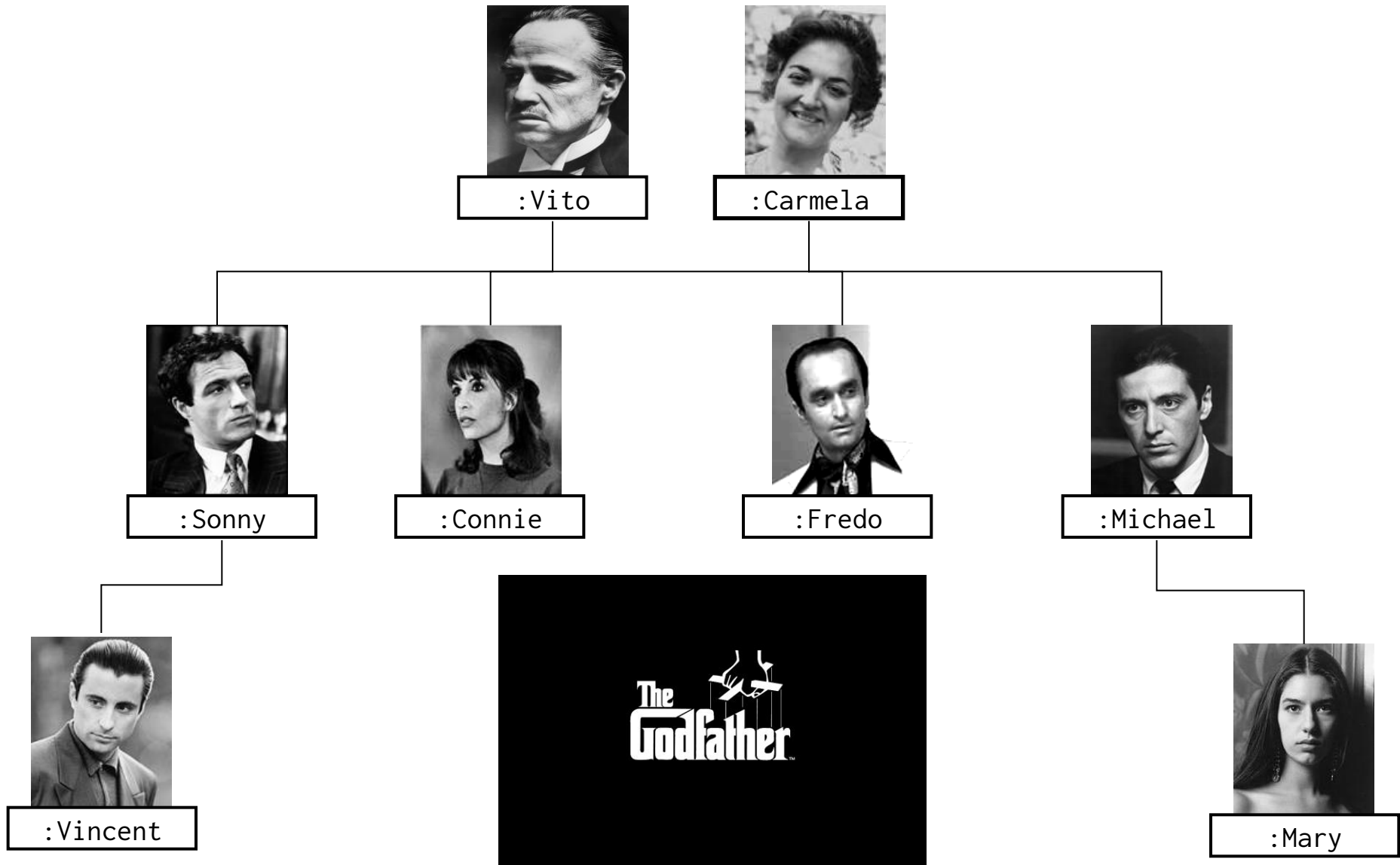


The image features a large iceberg floating in a blue ocean under a blue sky with light clouds. The tip of the iceberg is above the water line, while the much larger, submerged part is below. In the bottom right corner, there is a cartoon illustration of an owl wearing a blue diving mask and a snorkel. The text '← RDFS' is in the upper right, and '← OWL' is in the middle right, both in a light blue color.

← RDFS

← OWL

# FAMILY RELATIONS IN OWL



TODAY'S TOPIC

# AN ONTOLOGY IS JUST SOME DEFINITIONS ...

```
:hasDaughter rdfs:subPropertyOf :hasChild .  
:hasChild rdf:type owl:AsymmetricProperty ; owl:inverseOf :hasParent ;  
    rdfs:subPropertyOf :ancestorOf .  
:ancestorOf rdf:type owl:TransitiveProperty .  
ex:Carmela :hasChild ex:Michael .  
ex:Michael :hasDaughter ex:Mary .  
ex:Mary a :Person .  
:Person owl:equivalentClass  
    [ owl:qualifiedCardinality 2 ;  
      owl:onProperty :hasParent ;  
      owl:onClass :Person ] .
```

... BUT WHAT DO THEY MEAN?

... AND WHAT CAN WE DO WITH ONTOLOGIES?

# CAPTURE THE MAIN IDEAS OF OWL

Fig 1. OWL according to the standard



Fig 2. OWL according to this class





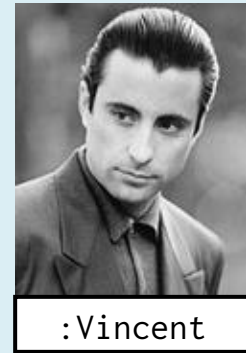
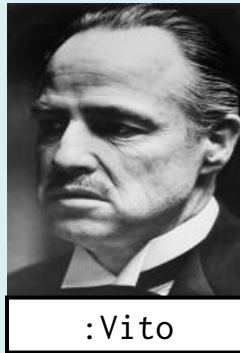
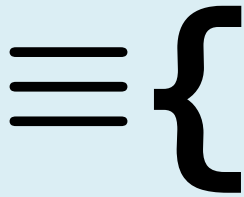


DEFINING OWL

# owl:oneOf ({})



```
:Godfather owl:equivalentClass [ owl:oneOf (:Vito :Michael :Vincent) ]
```



How can we formally define what this means?



## owl:oneOf ({})

```
:Godfather owl:equivalentClass [ owl:oneOf (:Vito :Michael :Vincent) ]
```



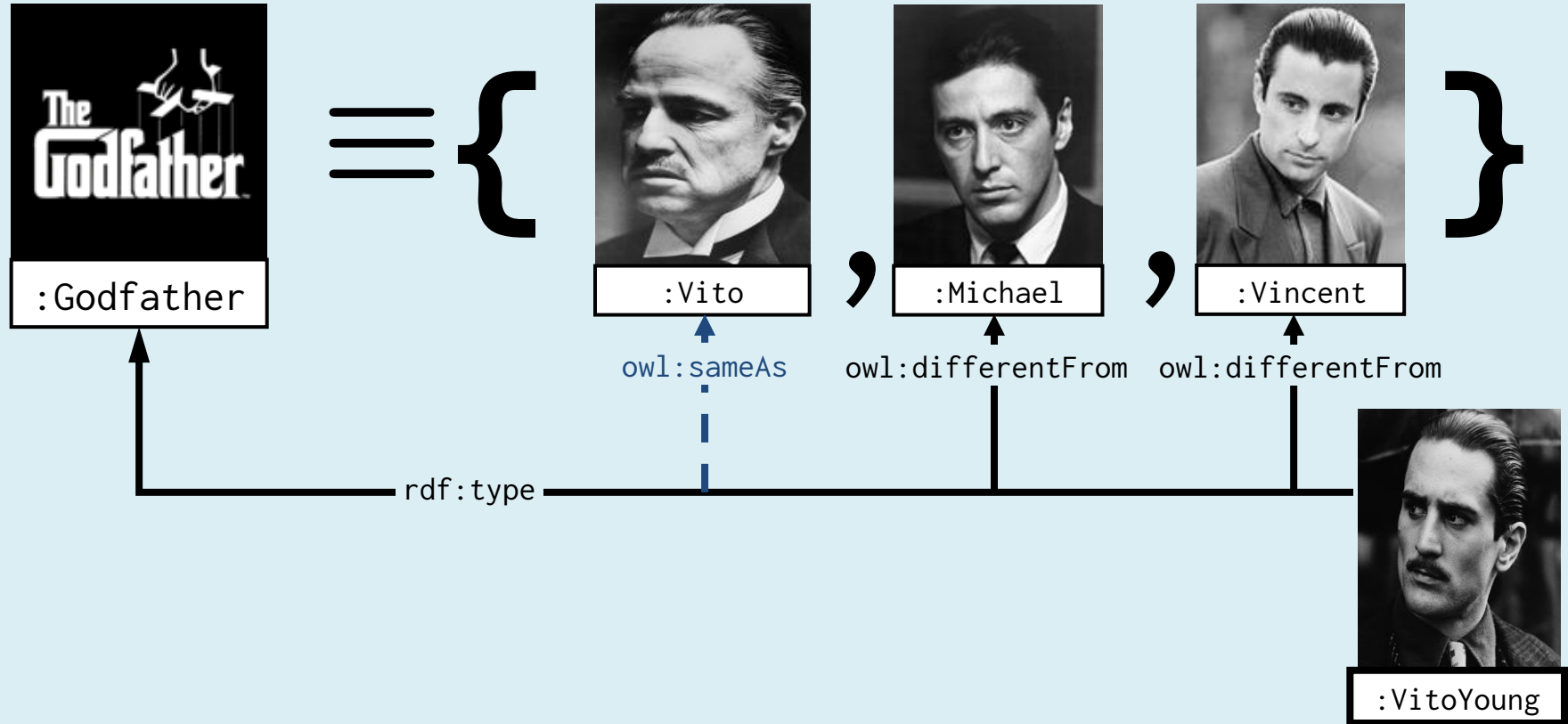
We could try with rules like:

```
?c owl:equivalentClass [ owl:oneOf (?x1 <...> ?xn) ]  
→ ?x1 a ?c . <...> ?xn a ?c .
```

# owl:oneOf ({})



```
:Godfather owl:equivalentClass [ owl:oneOf (:Vito :Michael :Vincent) ]
```



How do we define a rule for this case?

We could try a rule like:

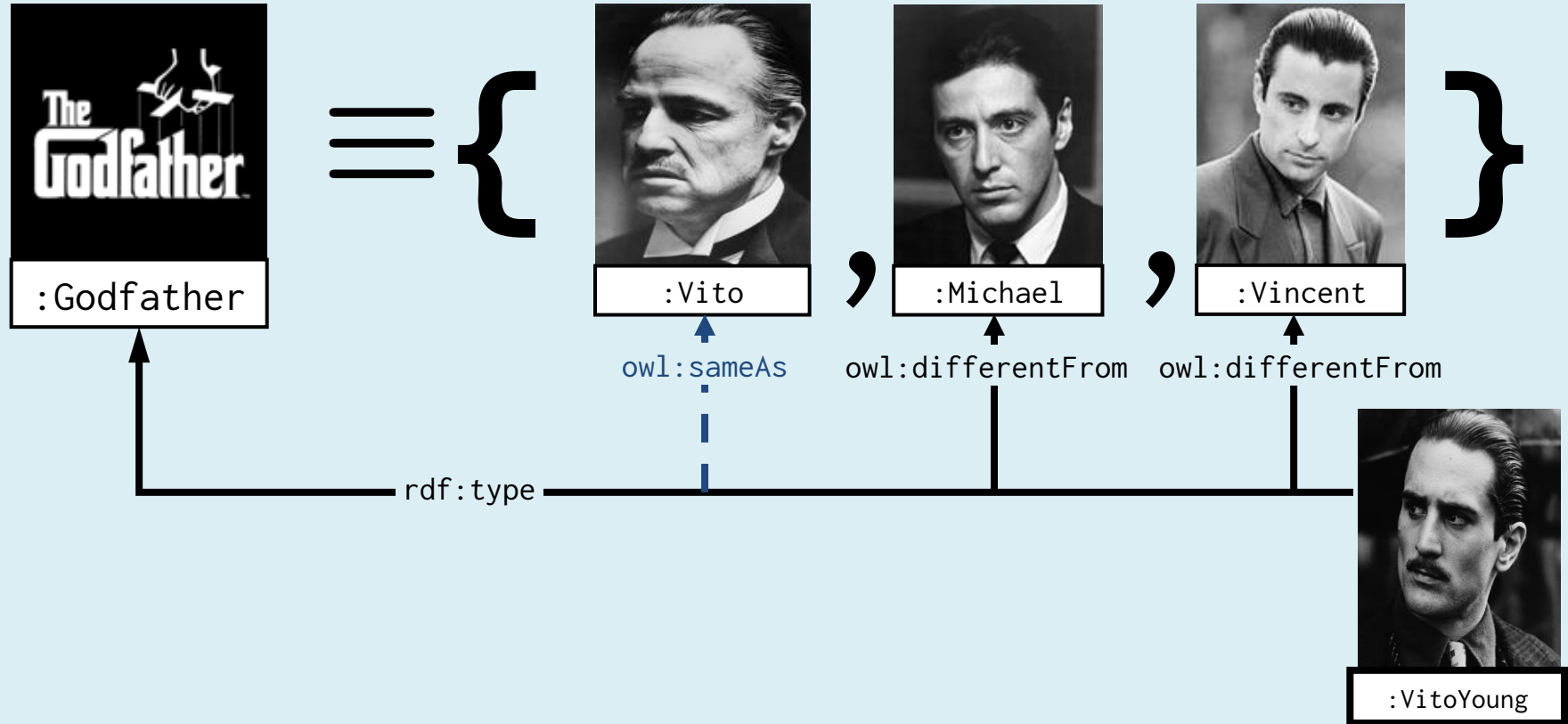
```
?c owl:equivalentClass [ owl:oneOf (?x1 <...> ?xn) ] . ?x a ?c .  
?x owl:differentFrom ?x2 , <...> , ?xn . → ?x owl:sameAs ?x1 .
```



# owl:oneOf ({})



```
:Godfather owl:equivalentClass [ owl:oneOf (:Vito :Michael :Vincent) ]
```



How do we define a rule for this case?

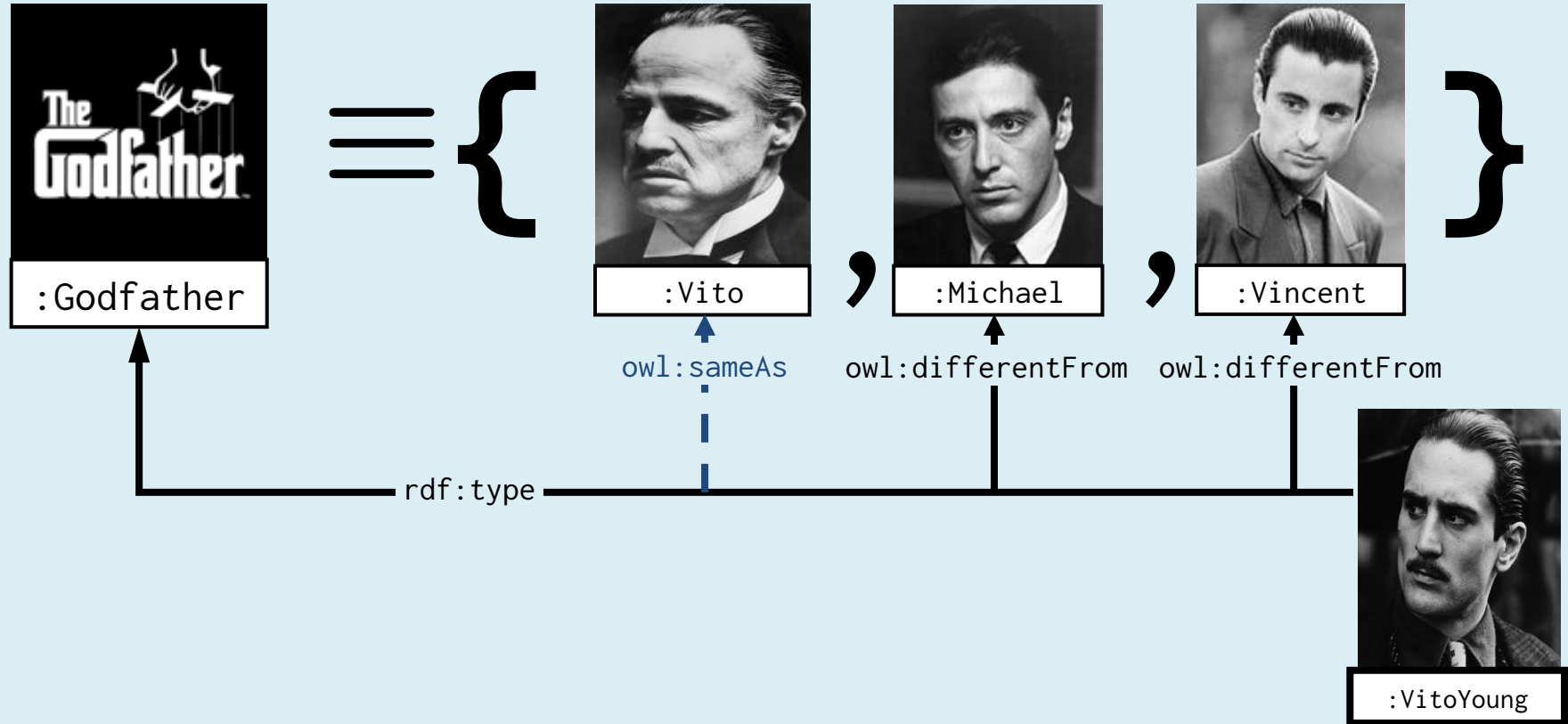
What if it's not the first element?

Have we considered all possible cases like this?

# owl:oneOf ({})



```
:Godfather owl:equivalentClass [ owl:oneOf (:Vito :Michael :Vincent) ]
```



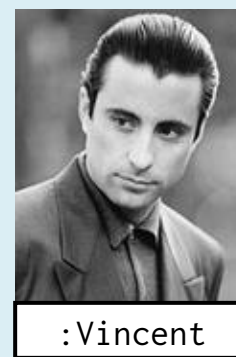
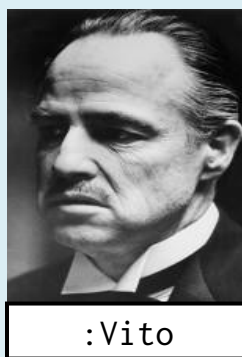
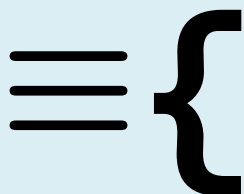
How do we define a rule for this case?

A finite set of rules may not be able to define everything we need in OWL!



## owl:oneOf ({})

```
:Godfather owl:equivalentClass [ owl:oneOf (:Vito :Michael :Vincent) ]
```



How can we begin to formally define what classes mean?

### Use set theory:

There is a set of individuals (like , etc.).

There is a set of classes (like , etc.)

Classes map to sets of individuals (like   $\mapsto$  { , ,  } )

Names map to individuals (like :Vito  $\mapsto$  )

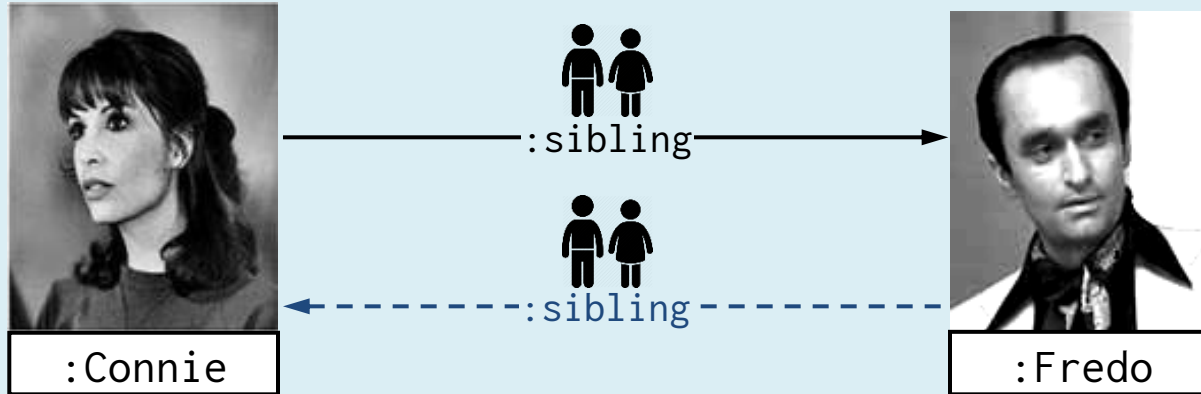
Other names map to classes (like :Godfather  $\mapsto$  )



# owl:SymmetricProperty



`:sibling rdf:type owl:SymmetricProperty .`



How can we begin to formally define what properties mean?

## Extend the set theory:

There is a set of individuals. Classes map to sets of individuals.

There is a set of properties (like , etc.).

Properties map to sets of pairs of individuals (like   $\mapsto \{ ( \text{Connie} , \text{Fredo} ) , \dots \}$ )

Names map to individuals, classes and or properties (like `:sibling`  $\mapsto$  )

# THE UNIVERSE



TO SIMPLIFY MATTERS

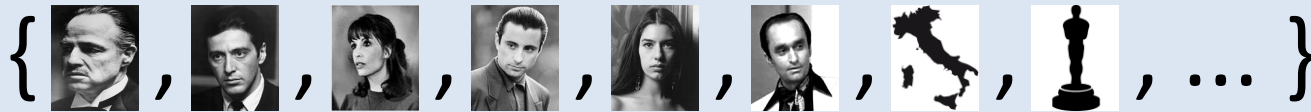
Define the universe as a set:

Set of everything:  $U$



# TO SIMPLIFY MATTERS

- Define the universe as a set:
  - Set of everything:  $\mathcal{U}$



What about classes and properties?

Classes are (sub)sets of  $\mathcal{U}$   
Properties are sets of pairs from  $\mathcal{U}$

VOCABULARY / DATA

# NEED TO NAME/DESCRIBE THINGS IN THE UNIVERSE

- Vocabulary (IRIs):  $V$
- Triples (RDF):  $V \times V \times V$ 
  - For brevity, we skip literals and blank nodes
- Graph (RDF):  $2^{V \times V \times V}$

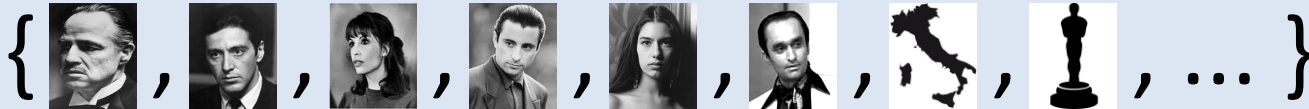
How do we relate the data to the universe?



INTERPRETATIONS

# INTERPRETATION OF A VOCABULARY

- Interpretation  $\mathcal{I}$  of vocabulary  $\mathcal{V}$ :  $\mathcal{I}(\mathcal{V}) = (\mathcal{U}, \mathcal{I}, \mathcal{C}, \mathcal{P})$   
 $\mathcal{U}$ : Universe / Set of everything (in the interpretation)



# INTERPRETATION OF A VOCABULARY

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  - $\mathcal{U}$ : Universe / Set of everything (in the interpretation)
  - $\mathcal{I}$ : Maps a vocabulary term  $v \in \mathcal{V}$  to an element of  $\mathcal{U}$   
(interpretation of Individuals)

$\mathcal{I}(:\text{Mary}) =$  

$\mathcal{I}(:\text{Fredo}) =$  

$\mathcal{I}(:\text{Vito}) =$  

$\mathcal{I}(:\text{Italy}) =$  

$\mathcal{I}(:\text{Oscar}) =$  

$\mathcal{I}(:\text{Sonny}) =$  

...

# INTERPRETATION OF A VOCABULARY

- Interpretation  $\mathcal{I}$  of vocabulary  $\mathcal{V}$ :  $\mathcal{I}(\mathcal{V}) = (\mathcal{U}, \mathcal{I}, \mathcal{C}, \mathcal{P})$ 
  - $\mathcal{U}$ : Universe / Set of everything (in the interpretation)
  - $\mathcal{I}$ : Maps a vocabulary term  $v \in \mathcal{V}$  to an element of  $\mathcal{U}$   
(interpretation of Individuals)
  - $\mathcal{C}$ : Maps a vocabulary term  $v \in \mathcal{V}$  to a subset of  $\mathcal{U}$   
(interpretation of Classes)

$\mathcal{C}(:\text{Person}) = \{ \text{img1}, \text{img2}, \text{img3}, \text{img4}, \text{img5}, \text{img6}, \text{img7}, \dots \}$

$\mathcal{C}(:\text{Godfather}) = \{ \text{img1}, \text{img2}, \text{img4}, \dots \}$

...

# INTERPRETATION OF A VOCABULARY

- Interpretation  $\mathcal{I}$  of vocabulary  $\mathcal{V}$ :  $\mathcal{I}(\mathcal{V}) = (\mathcal{U}, \mathcal{I}, \mathcal{C}, \mathcal{P})$ 
  - $\mathcal{U}$ : Universe / Set of everything (in the interpretation)
  - $\mathcal{I}$ : Maps a vocabulary term  $v \in \mathcal{V}$  to an element of  $\mathcal{U}$   
(interpretation of **I**ndividuals)
  - $\mathcal{C}$ : Maps a vocabulary term  $v \in \mathcal{V}$  to a subset of  $\mathcal{U}$   
(interpretation of **C**lasses)
  - $\mathcal{P}$ : Maps a vocabulary term  $v \in \mathcal{V}$  to a set of pairs from  $\mathcal{U}$   
(interpretation of **P**roperties)

$\mathcal{P}(:\text{Parent}) = \{ ( \text{img1}, \text{img2} ), ( \text{img3}, \text{img4} ), \dots \}$

$\mathcal{P}(:\text{Sibling}) = \{ ( \text{img5}, \text{img6} ), ( \text{img7}, \text{img8} ), \dots \}$

...

# INTERPRETATION OF A VOCABULARY

- Interpretation  $\mathcal{I}$  of vocabulary  $V$ :  $\mathcal{I}(V) = (U, I, C, P)$ 
  - $U$ : Universe / Set of everything (in the interpretation)
  - $I$ : Maps a vocabulary term  $v \in V$  to an element of  $U$   
(interpretation of **I**ndividuals)
  - $C$ : Maps a vocabulary term  $v \in V$  to a subset of  $U$   
(interpretation of **C**lasses)
  - $P$ : Maps a vocabulary term  $v \in V$  to a set of pairs from  $U$   
(interpretation of **P**roperties)

How do we interpret data/graphs and not just terms?

MODELS

# MODELS

- Let  $G$  be an RDF graph using vocabulary  $V$
- Let  $\mathcal{I}$  be an interpretation of vocabulary  $V$ :
  - $\mathcal{I}(V) = (U, I, C, P)$
- $\mathcal{I}$  is a model of  $G$  if and only if:
  - If  $(s, p, o) \in G$  then  $(I(s), I(o)) \in P(p)$

How do we define the semantics of types?



## MODELS (WITH CLASSES/TYPE/SUBCLASS)

- Let  $G$  be an RDF graph using vocabulary  $V$
- Let  $\mathcal{I}$  be an interpretation of vocabulary  $V$ :
  - $\mathcal{I}(V) = (U, I, C, P)$
- $\mathcal{I}$  is a model of  $G$  if and only if:
  - If  $(s, p, o) \in G$  then  $(I(s), I(o)) \in P(p)$
  - If  $(I(s), I(o)) \in P(\text{rdf:type})$  then  $I(s) \in C(o)$

How do we define the semantics of sub-class?

# MODELS (WITH RDFS SEMANTICS)

- Let  $G$  be an RDF graph using vocabulary  $V$
- Let  $\mathcal{I}$  be an interpretation of vocabulary  $V$ :
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- $\mathcal{I}$  is a model of  $G$  if and only if:
  - If  $(s, p, o) \in G$  then  $(I(s), I(o)) \in P(p)$
  - If  $(I(s), I(o)) \in P(\text{rdf:type})$  then  $I(s) \in C(o)$
  - If  $(I(s), I(o)) \in P(\text{rdfs:subClassOf})$  then  $C(s) \subseteq C(o)$

How do we define the semantics of sub-property?

# MODELS (WITH RDFS SEMANTICS)

- Let  $G$  be an RDF graph using vocabulary  $V$
- Let  $\mathcal{I}$  be an interpretation of vocabulary  $V$ :
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  - If  $(I(s), I(o)) \in P(\text{rdf:type})$  then  $I(s) \in C(o)$
  - If  $(I(s), I(o)) \in P(\text{rdfs:subClassOf})$  then  $C(s) \subseteq C(o)$
  - If  $(I(s), I(o)) \in P(\text{rdfs:subPropertyOf})$  then  $P(s) \subseteq P(o)$

How do we define the semantics of domain/range?

# MODELS (WITH RDFS SEMANTICS)

- Let  $G$  be an RDF graph using vocabulary  $V$
- Let  $\mathcal{I}$  be an interpretation of vocabulary  $V$ :
  - $\mathcal{I}(V) = (U, I, C, P)$
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  - If  $(I(s), I(o)) \in P(\text{rdf:type})$  then  $I(s) \in C(o)$
  - If  $(I(s), I(o)) \in P(\text{rdfs:subClassOf})$  then  $C(s) \subseteq C(o)$
  - If  $(I(s), I(o)) \in P(\text{rdfs:subPropertyOf})$  then  $P(s) \subseteq P(o)$
  - If  $(I(s), I(o)) \in P(\text{rdfs:domain})$  then  $\pi_1(P(s)) \subseteq C(o)$
  - If  $(I(s), I(o)) \in P(\text{rdfs:range})$  then  $\pi_2(P(s)) \subseteq C(o)$

# MODELS (WITH OWL SEMANTICS)

- $\mathcal{I}$  is a model of  $G$  if and only if:
  - If  $(s, p, o) \in G$  then  $(I(s), I(o)) \in P(p)$
  - If  $(I(s), I(o)) \in P(\text{rdf:type})$  then  $I(s) \in C(o)$
  - If  $(I(s), I(o)) \in P(\text{rdfs:subClassOf})$  then  $C(s) \subseteq C(o)$
  - If  $(I(s), I(o)) \in P(\text{rdfs:subPropertyOf})$  then  $P(s) \subseteq P(o)$
  - If  $(I(s), I(o)) \in P(\text{rdfs:domain})$  then  $\pi_1(P(s)) \subseteq C(o)$
  - If  $(I(s), I(o)) \in P(\text{rdfs:range})$  then  $\pi_2(P(s)) \subseteq C(o)$

What about OWL semantics?

How do we define the semantics of same-as/different-from?

# MODELS (WITH OWL SEMANTICS)

- $\mathcal{I}$  is a model of  $G$  if and only if:
  - If  $(s, p, o) \in G$  then  $(I(s), I(o)) \in P(p)$
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  - If  $(I(s), I(o)) \in P(\text{rdfs:subClassOf})$  then  $C(s) \subseteq C(o)$
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  - If  $(I(s), I(o)) \in P(\text{rdfs:range})$  then  $\pi_2(P(s)) \subseteq C(o)$
  - If  $(I(s), I(o)) \in P(\text{owl:sameAs})$  then  $I(s) = I(o)$
  - If  $(I(s), I(o)) \in P(\text{owl:differentFrom})$  then  $I(s) \neq I(o)$

How do we define inverse properties?

# MODELS (WITH OWL SEMANTICS)

- $\mathcal{I}$  is a model of  $G$  if and only if:
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  - If  $(I(s), I(o)) \in P(\text{rdf:type})$  then  $I(s) \in C(o)$
  - If  $(I(s), I(o)) \in P(\text{rdfs:subClassOf})$  then  $C(s) \subseteq C(o)$
  - If  $(I(s), I(o)) \in P(\text{rdfs:subPropertyOf})$  then  $P(s) \subseteq P(o)$
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  - If  $(I(s), I(o)) \in P(\text{owl:sameAs})$  then  $I(s) = I(o)$
  - If  $(I(s), I(o)) \in P(\text{owl:differentFrom})$  then  $I(s) \neq I(o)$
  - If  $(I(s), I(o)) \in P(\text{owl:inverseOf})$  then  $\forall x, y : (x, y) \in P(s)$   
if and only if  $(y, x) \in P(o)$

How do we define transitive properties?

# MODELS (WITH OWL SEMANTICS)

- $\mathcal{I}$  is a model of  $G$  if and only if:
  - If  $(s, p, o) \in G$  then  $(I(s), I(o)) \in P(p)$
  - If  $(I(s), I(o)) \in P(\text{rdf:type})$  then  $I(s) \in C(o)$
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  - If  $(I(s), I(o)) \in P(\text{owl:inverseOf})$  then  $\forall x, y : (x, y) \in P(s)$   
if and only if  $(y, x) \in P(o)$
  - If  $I(s) \in C(\text{owl:TransitiveProperty})$  then  $\forall x, y, z : (x, y) \in P(s)$  and  
 $(y, z) \in P(s)$  imply  $(x, z) \in P(s)$

How do we define disjoint classes?



# MODELS (WITH OWL SEMANTICS)

- $\mathcal{I}$  is a model of  $G$  if and only if:
  - If  $(s, p, o) \in G$  then  $(I(s), I(o)) \in P(p)$
  - If  $(I(s), I(o)) \in P(\text{rdf:type})$  then  $I(s) \in C(o)$
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  - If  $(I(s), I(o)) \in P(\text{owl:sameAs})$  then  $I(s) = I(o)$
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if and only if  $(y, x) \in P(o)$
  - If  $I(s) \in C(\text{owl:TransitiveProperty})$  then  $\forall x, y, z : (x, y) \in P(s)$  and  
 $(y, z) \in P(s)$  imply  $(x, z) \in P(s)$
  - If  $(I(s), I(o)) \in P(\text{owl:disjointWith})$  then  $C(s) \cap C(o) = \emptyset$

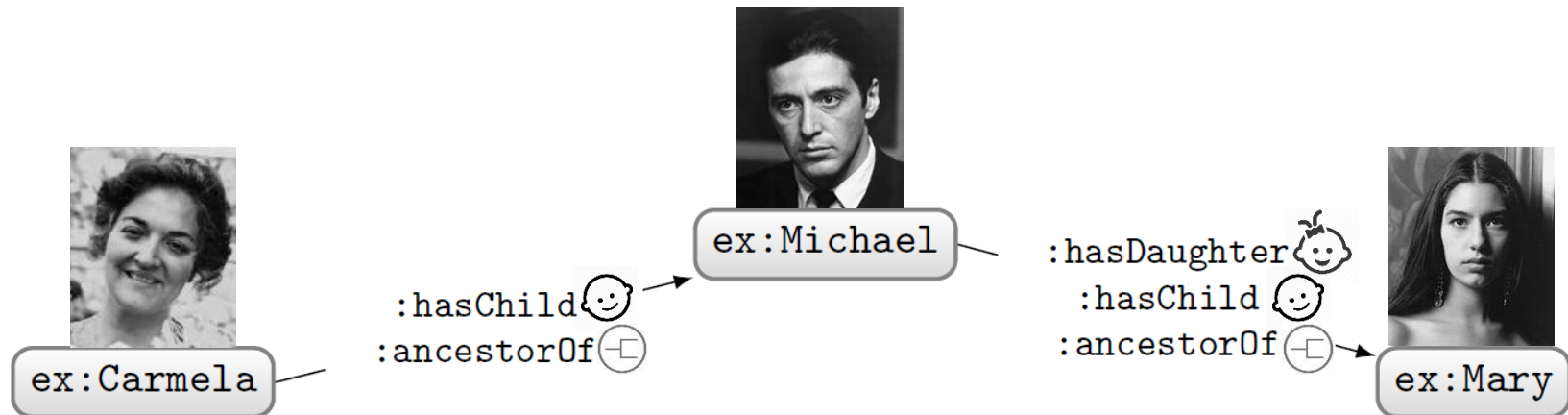
... and so on.

(BACK TO) MODELS

# MODELS OF GRAPHS/ONTOLOGIES

A model is any world that an ontology might describe

```
:hasDaughter rdfs:subPropertyOf :hasChild .  
:hasChild rdfs:subPropertyOf :ancestorOf .  
:ancestorOf rdf:type owl:TransitiveProperty .  
ex:Carmela :hasChild ex:Michael .  
ex:Michael :hasDaughter ex:Mary .
```



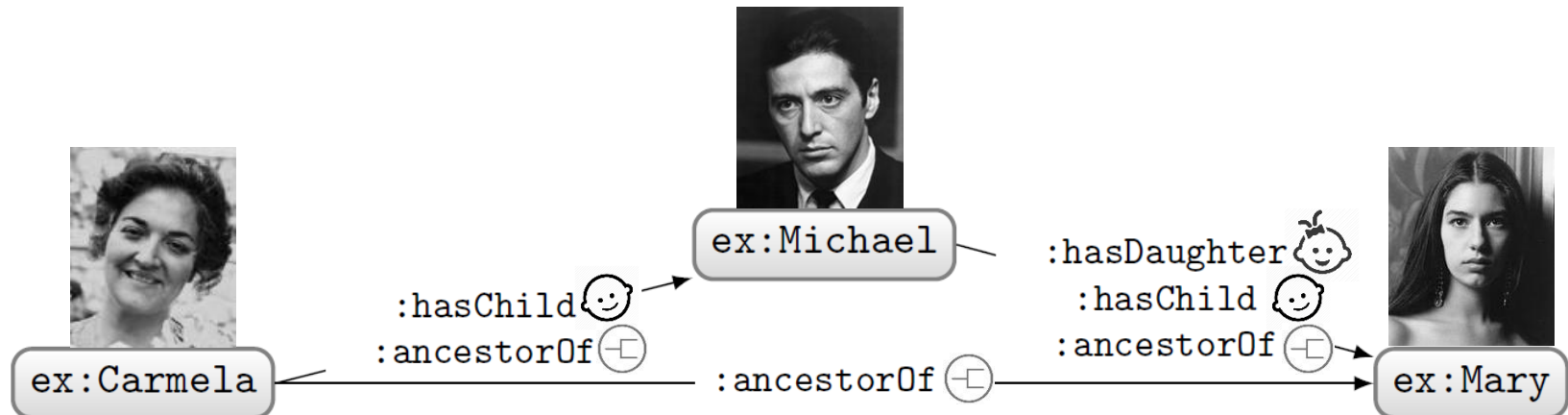
Is this a model of the ontology?

No, since  (ex:Carmela) needs to be an  (:ancestorOf)  (ex:Mary)

# MODELS OF ONTOLOGIES

A model is any world that an ontology might describe

```
:hasDaughter rdfs:subPropertyOf :hasChild .  
:hasChild rdfs:subPropertyOf :ancestorOf .  
:ancestorOf rdf:type owl:TransitiveProperty .  
ex:Carmela :hasChild ex:Michael .  
ex:Michael :hasDaughter ex:Mary .
```



Now we have a model of the ontology (for the given semantics).

# MODELS OF ONTOLOGIES

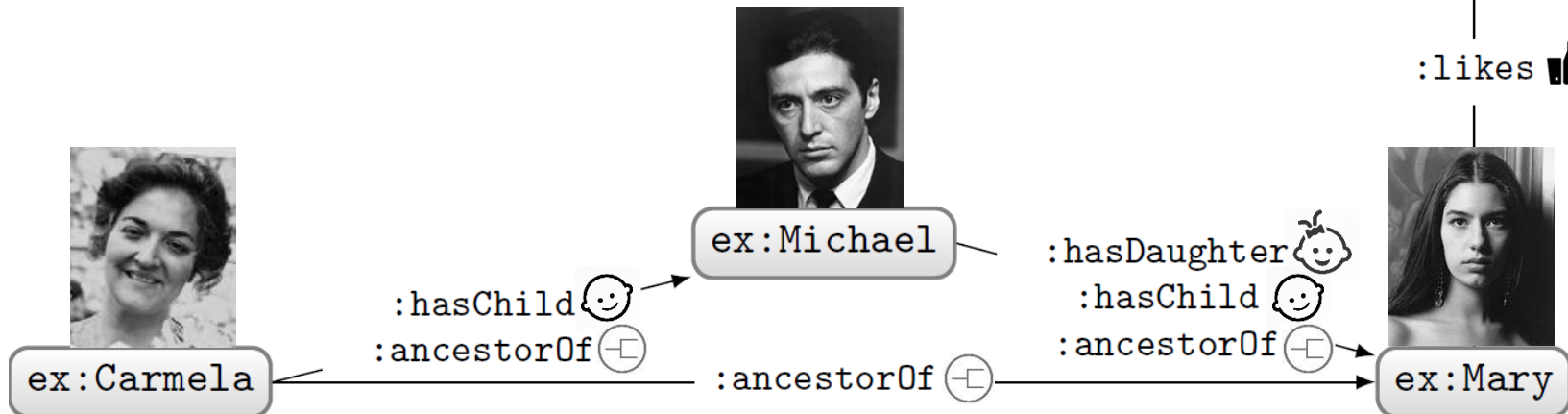
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:hasChild rdfs:subPropertyOf :ancestorOf .  
:ancestorOf rdf:type owl:TransitiveProperty .  
ex:Carmela :hasChild ex:Michael .  
ex:Michael :hasDaughter ex:Mary .
```



ex:Cake

:likes 👍



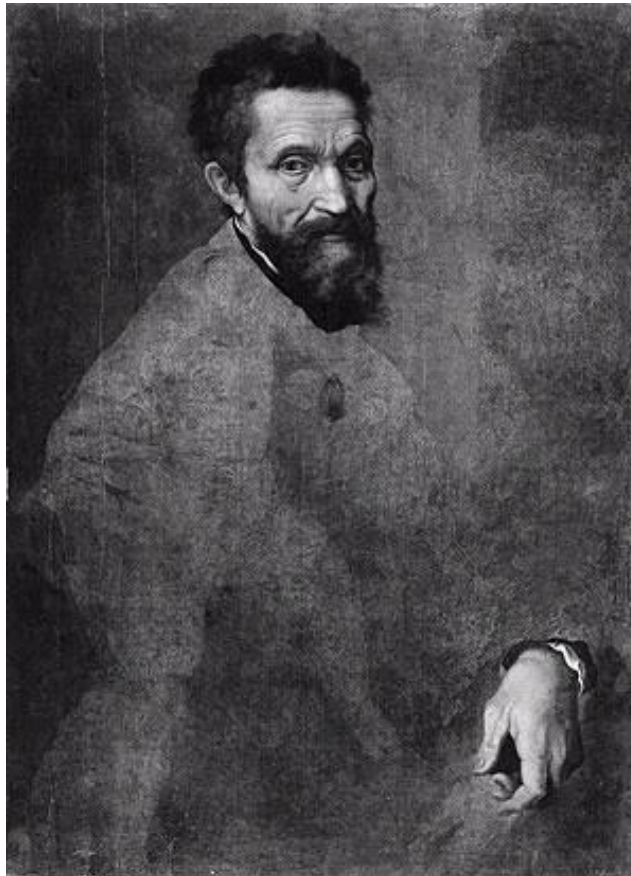
Is this a model of the ontology?

This is also a model (given the Open World Assumption)!

ENTAILMENT ...

# AXIOMS REMOVE MODELS

- At the start (almost) any world is possible!
- As we add axioms, fewer worlds are possible



The sculpture is already complete within the marble block, before I start my work. It is already there, I just have to chisel away the superfluous material.



# ONTOLOGY $O$ ENTAILS $O'$ ( $O \models O'$ )

```
:hasDaughter rdfs:subPropertyOf :hasChild .  
:hasChild owl:inverseOf :hasParent ; rdfs:subPropertyOf :ancestorOf .  
:ancestorOf rdf:type owl:TransitiveProperty .  
ex:Carmela :hasChild ex:Michael .  
ex:Michael :hasDaughter ex:Mary .
```

```
ex:Michael :hasParent ex:Carmela .  
ex:Michael :hasChild ex:Mary .  
ex:Carmela :ancestorOf ex:Mary .
```

Any model of  $O$  is a model of  $O'$

( $O'$  forms part of the models of  $O$ )

( $O'$  says nothing new over  $O$ )



ENTAILMENT SYMBOL:  $\models$

$$O \models O'$$

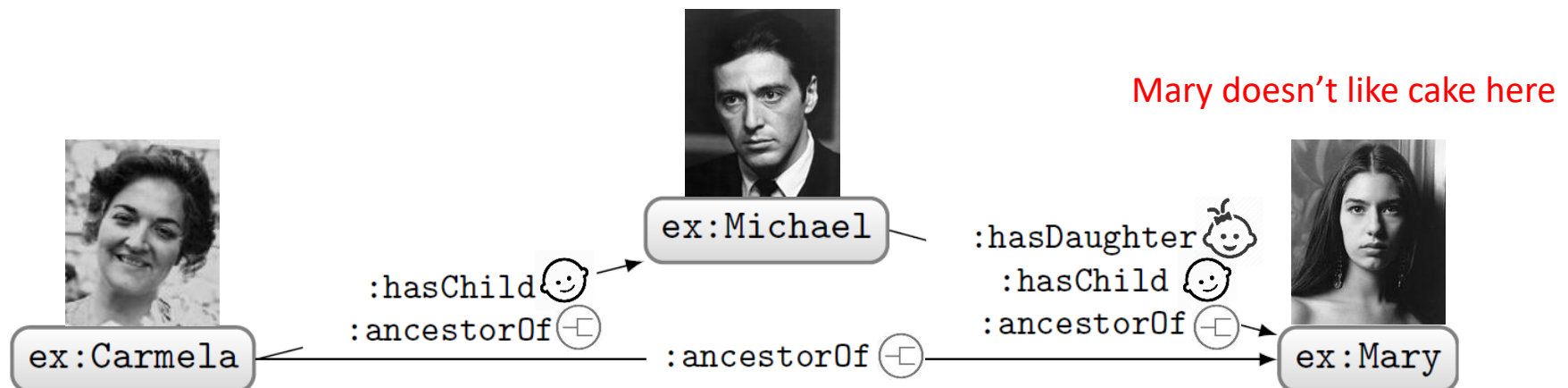
# ONTOLOGY ENTAILMENT

```
ex:Carmela :hasChild ex:Michael .  
ex:Michael :hasChild ex:Mary .  
ex:Carmela :ancestorOf ex:Mary .
```

```
ex:Carmela :ancestorOf ex:Mary .  
ex:Mary :likes ex:Cake .
```

Does  $O$  entail  $O'$  ( $O \models O'$ )?

**No!** There are models of  $O$  that are not models of  $O'$  ...



REASONING TASKS ...

# MATERIALISATION: WRITE DOWN ENTAILMENTS

```
:hasDaughter rdfs:subPropertyOf :hasChild .  
:hasChild rdf:type owl:AsymmetricProperty ; owl:inverseOf :hasParent ;  
  rdfs:subPropertyOf :ancestorOf .  
:ancestorOf rdf:type owl:TransitiveProperty .  
ex:Carmela :hasChild ex:Michael .  
ex:Michael :hasDaughter ex:Mary .  
ex:Mary a :Person .  
:Person owl:equivalentClass  
  [ owl:qualifiedCardinality 2 ;  
    owl:onProperty :hasParent ;  
    owl:onClass :Person ] .
```

```
ex:Michael :hasParent ex:Carmela .  
ex:Michael :hasChild ex:Mary .  
ex:Carmela :ancestorOf ex:Mary .  
...
```

Any problems with this?

# MATERIALISATION: WRITE DOWN ENTAILMENTS

```
:hasDaughter rdfs:subPropertyOf :hasChild .
:hasChild rdf:type owl:AsymmetricProperty ; owl:inverseOf :hasParent ;
  rdfs:subPropertyOf :ancestorOf .
:ancestorOf rdf:type owl:TransitiveProperty .
ex:Carmela :hasChild ex:Michael .
ex:Michael :hasDaughter ex:Mary .
ex:Mary a :Person .
:Person owl:equivalentClass
  [ owl:qualifiedCardinality 2 ;
    owl:onProperty :hasParent ;
    owl:onClass :Person ] .
```

```
ex:Michael :hasParent ex:Carmela .
ex:Michael :hasChild ex:Mary .
ex:Carmela :ancestorOf ex:Mary .
```

```
ex:Mary :hasParent _:parent1 . _:parent1 a :Person .
ex:Mary :hasParent _:parent2 . _:parent2 a :Person .
_:parent1 :hasParent _:parent11 . _:parent11 a :Person .
_:parent1 :hasParent _:parent12 . _:parent12 a :Person . ...
```

# MATERIALISATION: WRITE DOWN ENTAILMENTS

```
:hasDaughter rdfs:subPropertyOf :hasChild .
:hasChild rdf:type owl:AsymmetricProperty ; owl:inverseOf :hasParent ;
  rdfs:subPropertyOf :ancestorOf .
:ancestorOf rdf:type owl:TransitiveProperty .
ex:Carmela :hasChild ex:Michael .
ex:Michael :hasDaughter ex:Mary .
ex:Mary a :Person .
:Person owl:equivalentClass
  [ owl:qualifiedCardinality 2 ;
    owl:onProperty :hasParent ;
    owl:onClass :Person ] .
```

```
ex:Michael :hasParent ex:Carmela .
ex:Michael :hasChild ex:Mary .
ex:Carmela :ancestorOf ex:Mary .
```

```
:Person rdfs:subClassOf [ owl:maxQualifiedCardinality 2 ; owl:onProperty
  :hasParent ; owl:onClass :Person ] .
:Person rdfs:subClassOf [ owl:maxQualifiedCardinality 3 ; owl:onProperty
  :hasParent ; owl:onClass :Person ] . ...
```

# MATERIALISATION: WRITE DOWN ENTAILMENTS

```
:hasDaughter rdfs:subPropertyOf :hasChild .
:hasChild rdf:type owl:AsymmetricProperty ; owl:inverseOf :hasParent ;
  rdfs:subPropertyOf :ancestorOf .
:ancestorOf rdf:type owl:TransitiveProperty .
ex:Carmela :hasChild ex:Michael .
ex:Michael :hasDaughter ex:Mary .
ex:Mary a :Person .
:Person owl:equivalentClass
  [ owl:qualifiedCardinality 2 ;
    owl:onProperty :hasParent ;
    owl:onClass :Person ] .
```



```
ex:Michael :hasParent ex:Carmela .
ex:Michael :hasChild ex:Mary .
ex:Carmela :ancestorOf ex:Mary .
```

```
:Person rdfs:subClassOf [ owl:maxQualifiedCardinality 2 ; owl:onProperty
  :hasParent ; owl:onClass :Person ] .
:Person rdfs:subClassOf [ owl:maxQualifiedCardinality 3 ; owl:onProperty
  :hasParent ; owl:onClass :Person ] . ...
```

# ONTOLOGY SATISFIABILITY: DOES $\mathcal{O}$ HAVE A MODEL?

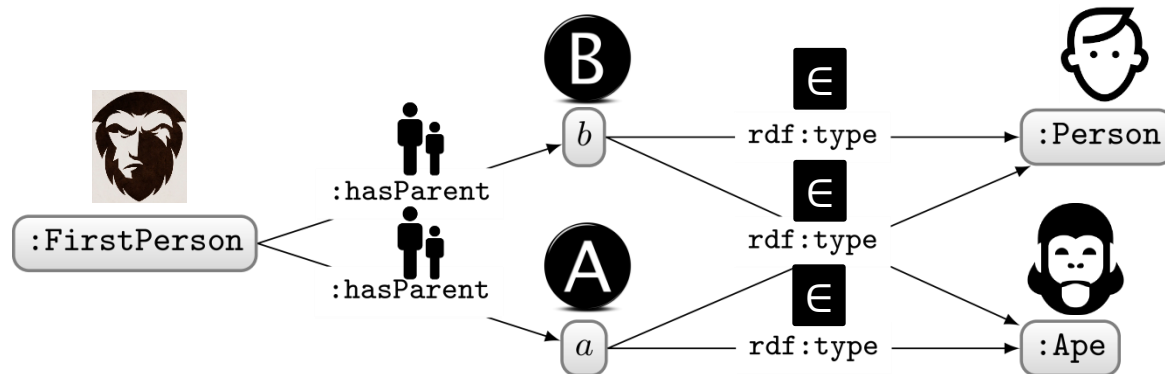
```
:Person owl:equivalentClass
```

```
[ owl:qualifiedCardinality 2 ; owl:onProperty :hasParent ;  
  owl:onClass :Person ] .
```

```
:FirstPerson a :Person ,
```

```
[ owl:qualifiedCardinality 2 ; owl:onProperty :hasParent ;  
  owl:onClass :Ape ] .
```

Does  $\mathcal{O}$  have any model?



YES! Ontology  $\mathcal{O}$  is **Satisfiable!**

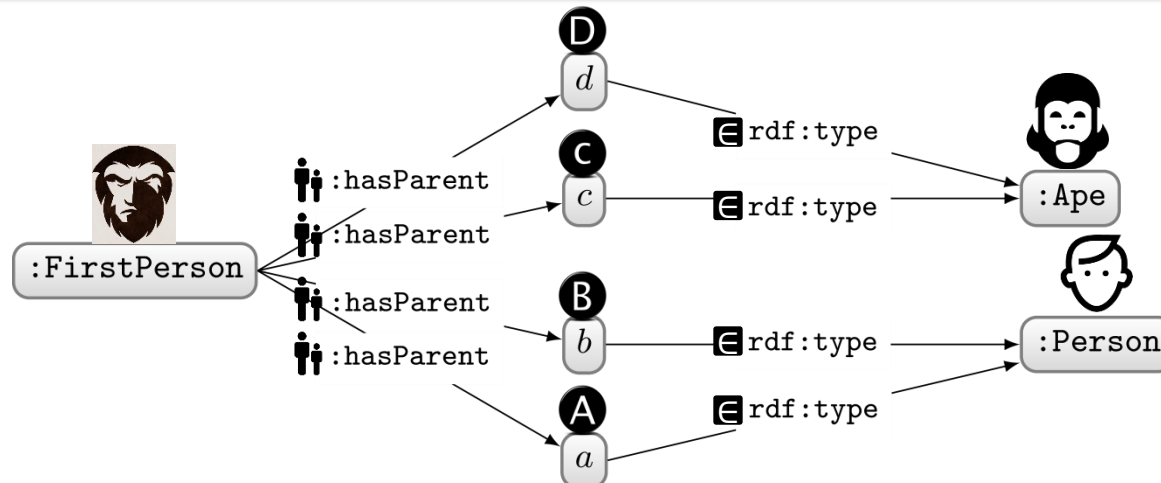


# ONTOLOGY SATISFIABILITY:

DOES **O** HAVE A “MODEL”?

```
:Person owl:equivalentClass  
  [ owl:qualifiedCardinality 2 ; owl:onProperty :hasParent ;  
    owl:onClass :Person ] .  
:FirstPerson a :Person ,  
  [ owl:qualifiedCardinality 2 ; owl:onProperty :hasParent ;  
    owl:onClass :Ape ] .  
:Ape owl:disjointWith :Person .
```

So does **O** have a model now?



YES! Ontology **O** is still **Satisfiable**!

# ONTOLOGY SATISFIABILITY:

DOES **O** HAVE A “MODEL”?

```
:Person owl:equivalentClass
  [ owl:qualifiedCardinality 2 ; owl:onProperty :hasParent ;
    owl:onClass :Person ] .
:FirstPerson a :Person ,
  [ owl:qualifiedCardinality 2 ; owl:onProperty :hasParent ;
    owl:onClass :Ape ] .
:Ape owl:disjointWith :Person .
```

What more would we have to add to **O** to make it **Unsatisfiable**?

```
:Person rdfs:subClassOf
  [ owl:cardinality 2 ; owl:onProperty :hasParent ] .
```

OR

```
:Person owl:equivalentClass
  [ owl:allValuesFrom :Person ; owl:onProperty :hasParent ] .
```

OR

```
:FirstPerson a :Ape .
```

OR

```
:FirstPerson a owl:Nothing .
```

OR ...

# ONTOLOGY SATISFIABILITY:

DOES **O** HAVE A “MODEL”?

```
:Person owl:equivalentClass
```

```
[ owl:qualifiedCardinality 2 ; owl:onProperty :hasParent ;
```

An unsatisfiable ontology cannot model any world!

It is inconsistent!



```
:FirstPerson a :Ape .
```

OR

OR

```
:FirstPerson a owl:Nothing .
```

OR ...

# ENTAILMENT CHECKING: DOES $O$ ENTAIL $O'$ ?

```
:hasDaughter rdfs:subPropertyOf :hasChild .  
:hasChild rdf:type owl:AsymmetricProperty ;  
  owl:inverseOf :hasParent ;  
  rdfs:subPropertyOf :ancestorOf .  
:ancestorOf rdf:type owl:TransitiveProperty .  
ex:Carmela :hasChild ex:Michael .  
ex:Michael :hasDaughter ex:Mary .
```

```
ex:Michael :hasParent ex:Carmela .  
ex:Michael :hasChild ex:Mary .  
ex:Carmela :ancestorOf ex:Mary .
```

Alternatively: Are all models of  $O$  models of  $O'$  too?

REASONING ...

# HOW CAN WE PERFORM REASONING?

- Does Ontology  $O$  entail  $O'$ ?

```
:hasDaughter rdfs:subPropertyOf :hasChild .  
ex:Michael :hasDaughter ex:Mary .
```

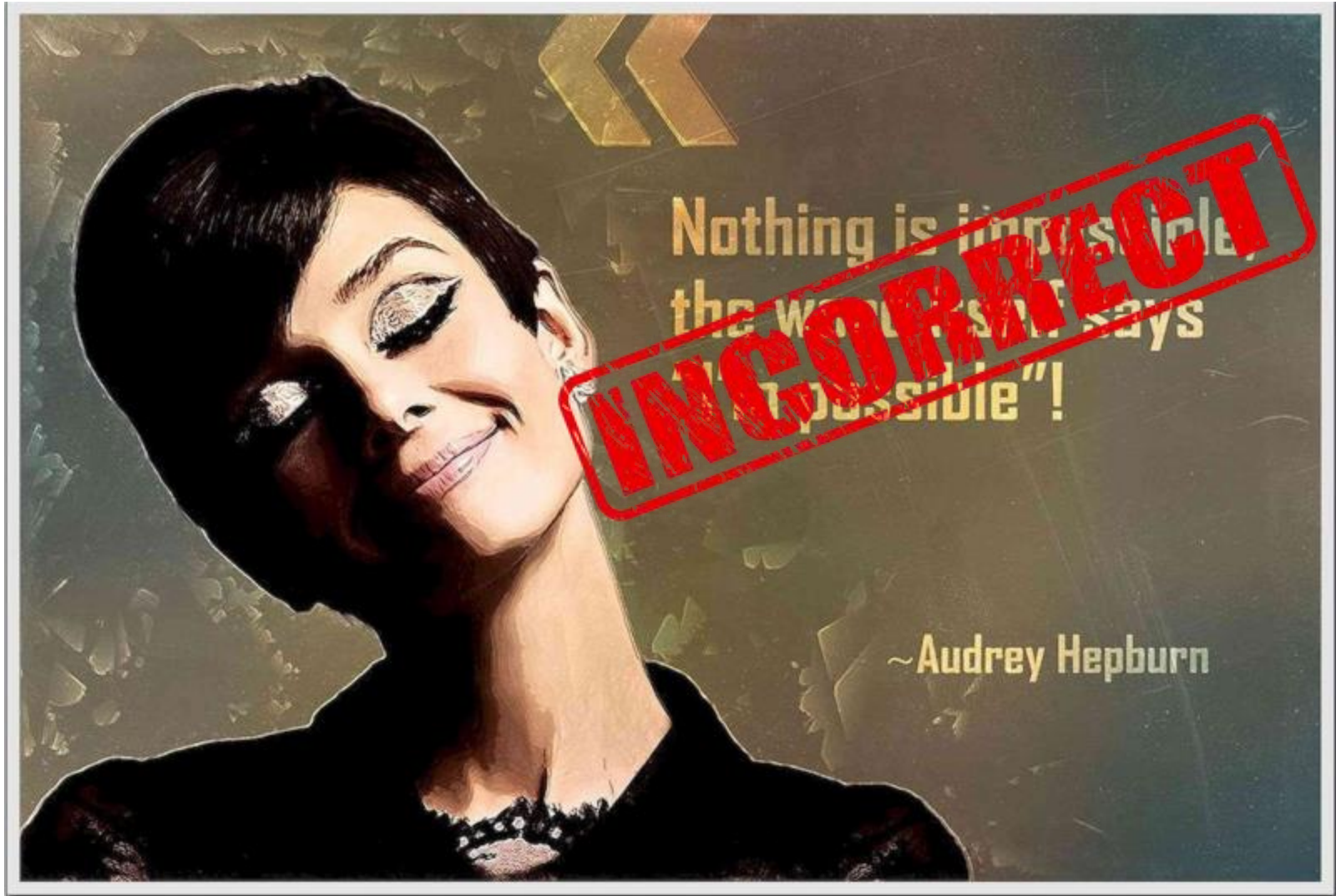
```
ex:Michael :hasChild ex:Mary .
```

- Could instead ask: is " $O \cup \neg O'$ " unsatisfiable?

```
:hasDaughter rdfs:subPropertyOf :hasChild .  
ex:Michael :hasDaughter ex:Mary .  
[] owl:sourceIndividual ex:Michael ;  
   owl:assertionProperty :hasChild ;  
   owl:targetIndividual ex:Mary .
```

Can reduce entailment to unsatisfiability!

SO HOW DO WE TEST UNSATISFIABILITY THEN?



UNDECIDABILITY ...



# A SIMPLE JAVA PROGRAM ...

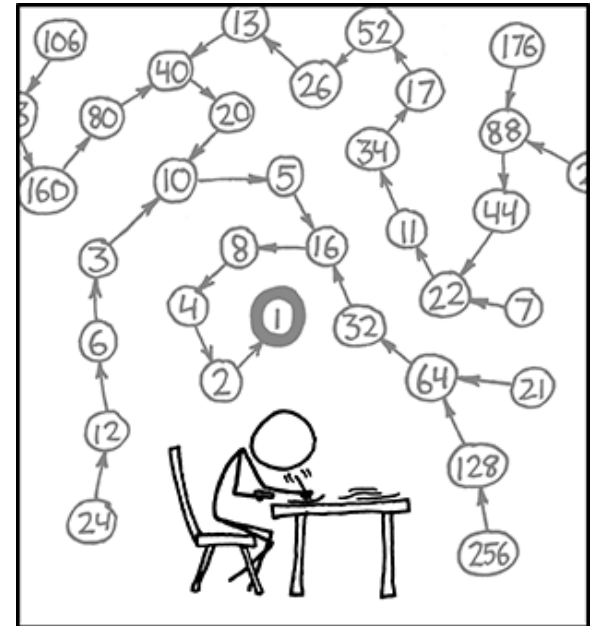
```
public class Collatz {  
    public static void collatz(int n) {  
        StdOut.print(n + " ");  
        if (n == 1) return;  
        else if (n % 2 == 0) collatz(n / 2);  
        else collatz(3*n + 1);  
    }  
  
    public static void main(String[] args) {  
        int N = Integer.parseInt(args[0]);  
        collatz(N);  
        StdOut.println();  
    }  
}
```

In: 6  
3  
10  
5  
16  
8  
4  
2  
1 (end)

Does this Java program terminate  
on all possible (positive) inputs?

# A SIMPLE JAVA PROGRAM ...

```
public class Collatz {  
    public static void collatz(int n) {  
        StdOut.print(n + " ");  
        if (n == 1) return;  
        else if (n % 2 == 0) collatz(n / 2);  
        else collatz(3*n + 1);  
    }  
  
    public static void main(String[] args) {  
        int N = Integer.parseInt(args[0]);  
        collatz(N);  
        StdOut.println();  
    }  
}
```



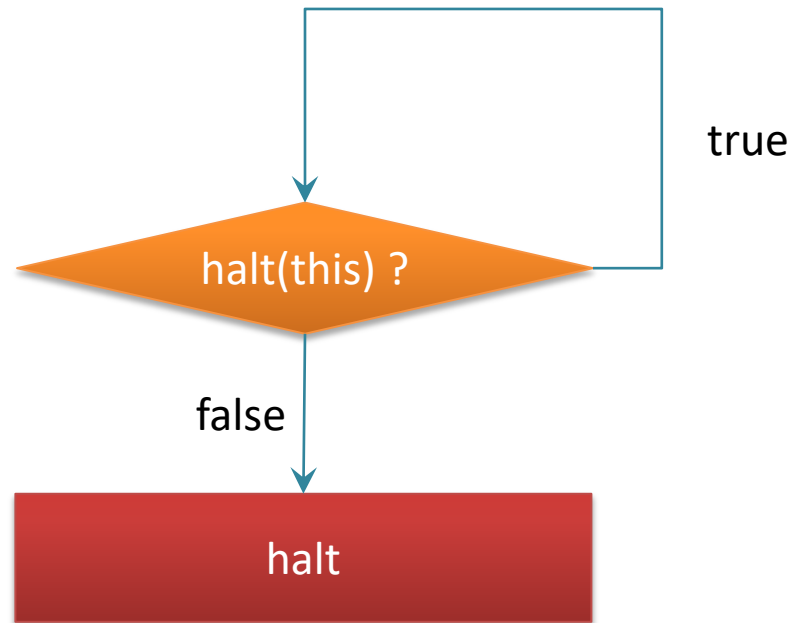
THE COLLATZ CONJECTURE STATES THAT IF YOU PICK A NUMBER, AND IF IT'S EVEN DIVIDE IT BY TWO AND IF IT'S ODD MULTIPLY IT BY THREE AND ADD ONE, AND YOU REPEAT THIS PROCEDURE LONG ENOUGH, EVENTUALLY YOUR FRIENDS WILL STOP CALLING TO SEE IF YOU WANT TO HANG OUT.

- **Collatz conjecture:** an unsolved problem in mathematics
- If we knew that this program terminates or does not terminate on all natural numbers (not just ints), this problem would be solved!

# HALTING PROBLEM

- **Input:** a program and an input to that program
- **Output:**
  - true if the program halts on that input
  - false otherwise
- **UNDECIDABLE:** a general algorithm to solve the Halting Problem does not exist!
  - It will not halt for all program–input pairs!
  - It may halt for some program–input pairs

# A QUICK SKETCH OF HALTING PROBLEM PROOF



## PROBLEM: CONSECUTIVE '1'S IN $\pi$

- **Input:** A natural number  $n$
- **Output:**
  - true if  $\pi$  contains  $n$  consecutive '1's
  - false otherwise

Is this problem **DECIDABLE** or **UNDECIDABLE**?

... i.e., does there exist a program that halts (with the correct answer) for all  $n$ ?

What if we knew the maximum sequence of consecutive '1's in  $\pi$ ?

**if** ( $n \leq \text{MAX}$ ) **return** true; **else return** false;

- there must exist a MAX sequence of consecutive '1's in  $\pi$  (even if it's  $\infty$ )  
∴ there must exist a correct program that halts (even if we don't know its details)  
∴ problem is **DECIDABLE**!

# PROBLEM: COLLATZ HALTING PROBLEM

- **Input:** [none]
- **Output:**
  - true if Collatz program halts on all inputs
  - false otherwise

Is this problem **DECIDABLE** or **UNDECIDABLE**?

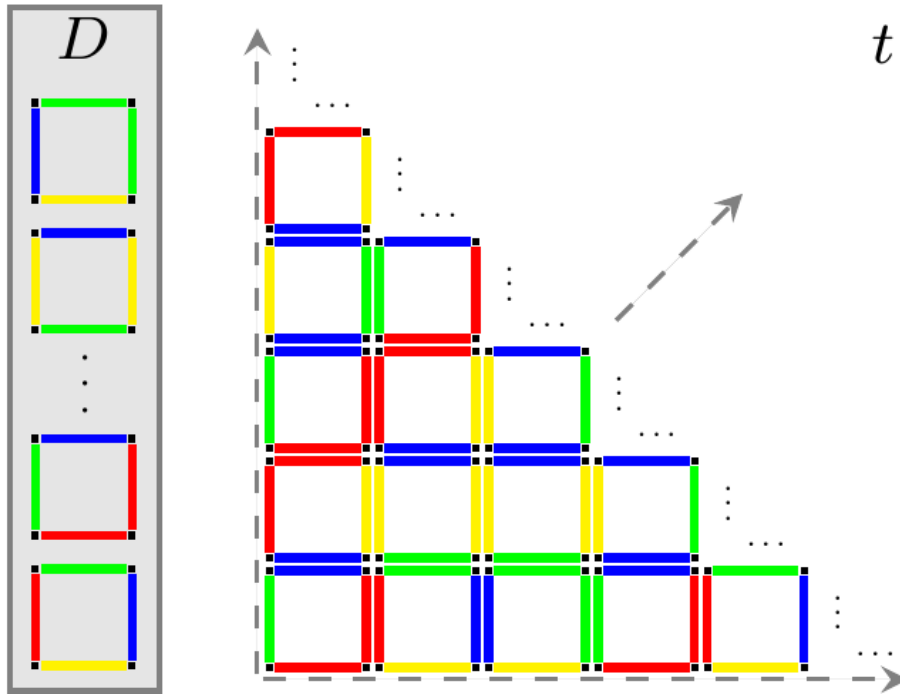
**(P1)** return true;

**(P2)** return false;

- either **(P1)** or **(P2)** must be correct
- ∴ there must exist a correct program that halts (even if we don't know it)  
∴ problem is **DECIDABLE**!

Halting Problem **UNDECIDABLE** in the general case  
(for all programs and inputs)

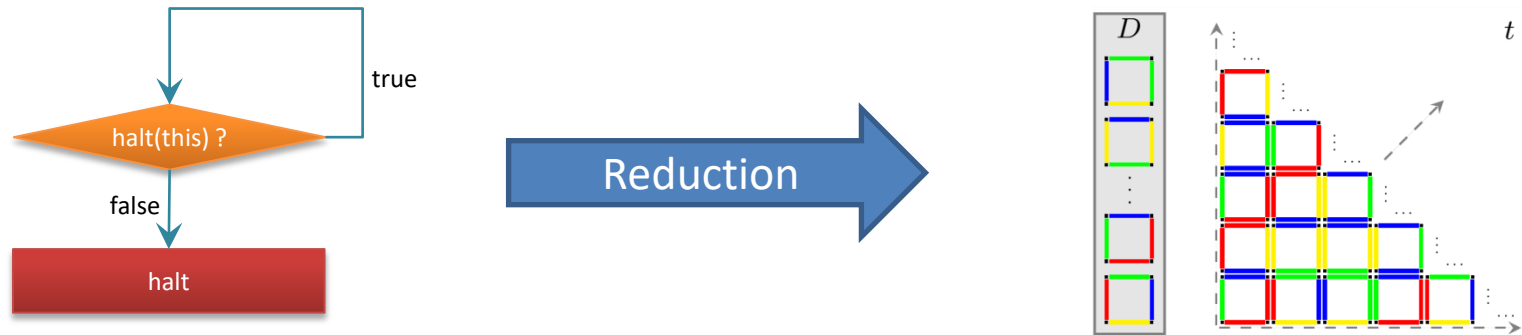
# DOMINO TILING PROBLEM



Is this problem  
**DECIDABLE** or  
**UNDECIDABLE**?

- **Input:** A set of Dominos (like  $D$ )
- **Output:**
  - true if there exists a valid infinite tiling (like  $t$ )
  - false otherwise

# CAN REDUCE FROM HALTING TO TILING



It has been shown that there exists a program that can reduce any Halting problem instance into a Domino Tiling problem instance

Now is the Domino Tiling problem **DECIDABLE** or **UNDECIDABLE**?

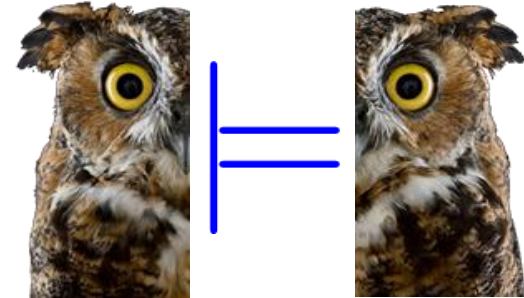
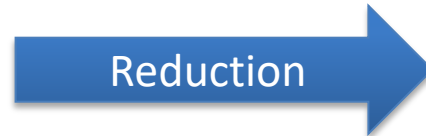
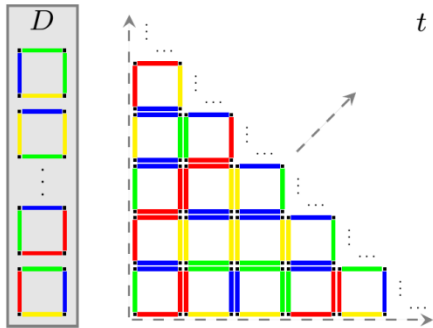
```
halts(p,in) {  
    D = reduce(p,in);  
    return hasTiling(D);  
}
```

- reduce(p, in) is **DECIDABLE**
- ∴ if hasTiling(D) were **DECIDABLE**, then halts(p, in) would be **DECIDABLE**
- but halts(p, in) is **UNDECIDABLE**
- ∴ hasTiling(D) must be **UNDECIDABLE**!

*If we have a decidable reduction from an **UNDECIDABLE** problem A to another problem B, then B must be **UNDECIDABLE***



# REDUCE FROM TILING TO OWL ENTAILMENT?



Does  $D$  have an infinite tiling?

Does OWL ontology  $O$  entail  $O'$ ?

How can we encode a Domino Tiling question into an OWL ontology entailment question?

**TO BE  
CONTINUED...** →

QUESTIONS?

