

**CC6202-1**

**LA WEB DE DATOS**

**PRIMAVERA 2016**

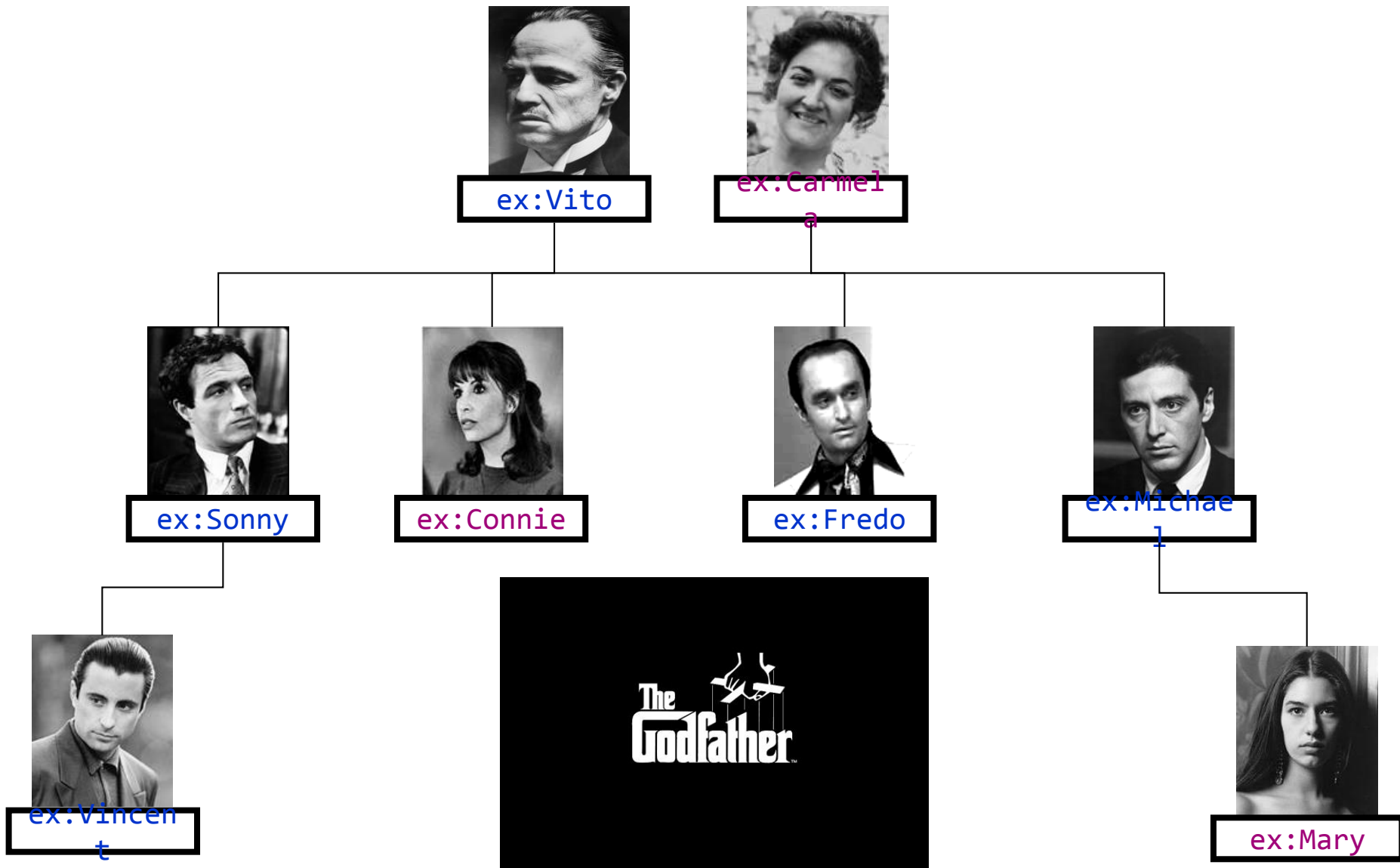
## **Lecture 6: Web Ontology Language (III)**

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**PREVIOUSLY ON  
“LA WEB DE DATOS”**

# Modelling family relations with OWL



# Materialisation:

## Write down entailments

```
:hasDaughter rdfs:subPropertyOf :hasChild .  
:hasChild rdf:type owl:AsymmetricProperty ; owl:inverseOf  
    :hasParent ;  
    rdfs:subPropertyOf :ancestorOf .  
:ancestorOf rdf:type owl:TransitiveProperty .  
ex:Carmela :hasChild ex:Michael .  
ex:Michael :hasDaughter ex:Mary .  
ex:Mary a :Person .  
:Person owl:equivalentClass  
    [ owl:qualifiedCardinality 2 ;  
      owl:onProperty :hasParent ;  
      owl:onClass :Person ] .
```

```
ex:Michael :hasParent ex:Carmela .  
ex:Michael :hasChild ex:Mary .  
ex:Carmela :ancestorOf ex:Mary .
```

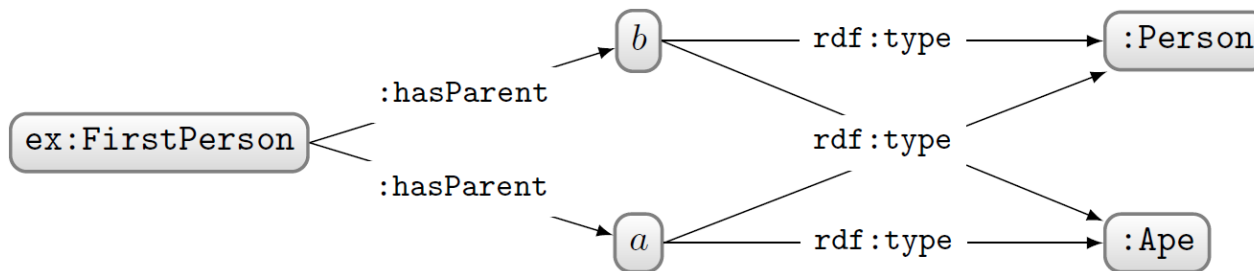
```
ex:Mary :hasParent _:parent1 . _:parent1 a :Person .  
ex:Mary :hasParent _:parent2 . _:parent2 a :Person .  
_:parent1 :hasParent _:parent11 . _:parent11 a :Person .  
_:parent2 :hasParent _:parent12 . _:parent12 a :Person . ...
```

# Ontology Satisfiability:

Does *O* have a “model”?

```
:Person owl:equivalentClass  
  [ owl:qualifiedCardinality 2 ; owl:onProperty :hasParent ;  
    owl:onClass :Person ] .  
:FirstPerson a :Person ,  
  [ owl:qualifiedCardinality 2 ; owl:onProperty :hasParent ;  
    owl:onClass :Ape ] .
```

So does *O* have a model?



YES! Ontology *O* is **Satisfiable**!

# Entailment checking:

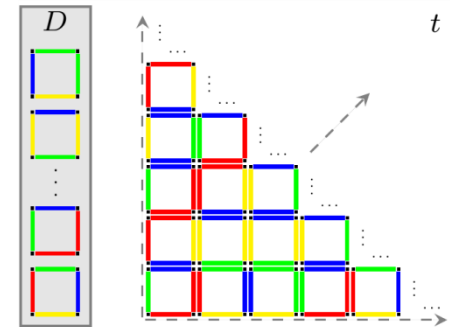
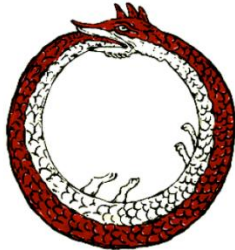
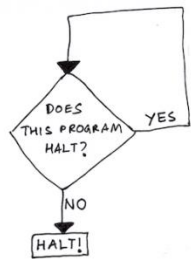
Does  $O$  entail  $O'$ ?

```
:hasDaughter rdfs:subPropertyOf :hasChild .  
:hasChild rdf:type owl:AsymmetricProperty ; owl:inverseOf  
  :hasParent ;  
  rdfs:subPropertyOf :ancestorOf .  
:ancestorOf rdf:type owl:TransitiveProperty .  
ex:Carmela :hasChild ex:Michael .  
ex:Michael :hasDaughter ex:Mary .
```

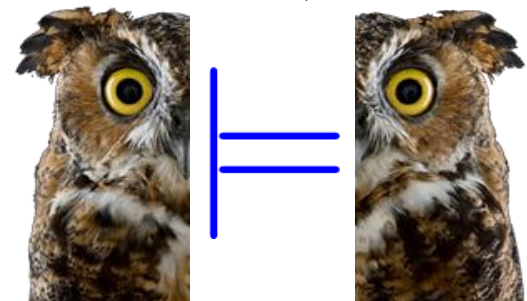
```
ex:Michael :hasParent ex:Carmela .  
ex:Michael :hasChild ex:Mary .  
ex:Carmela :ancestorOf ex:Mary .
```

Alternatively: Are all models of  $O$  models of  $O'$  too?

# OWL satisfiability/entailment is powerful



OWL satisfiability/entailment checking also **undecidable**!  
Otherwise could be used to solve Domino Tiling problem ...  
... and the Halting problem ...  
... and (given enough time), the Collatz conjecture ...  
... and a bunch of other stuff



**TODAY'S TOPIC ...**

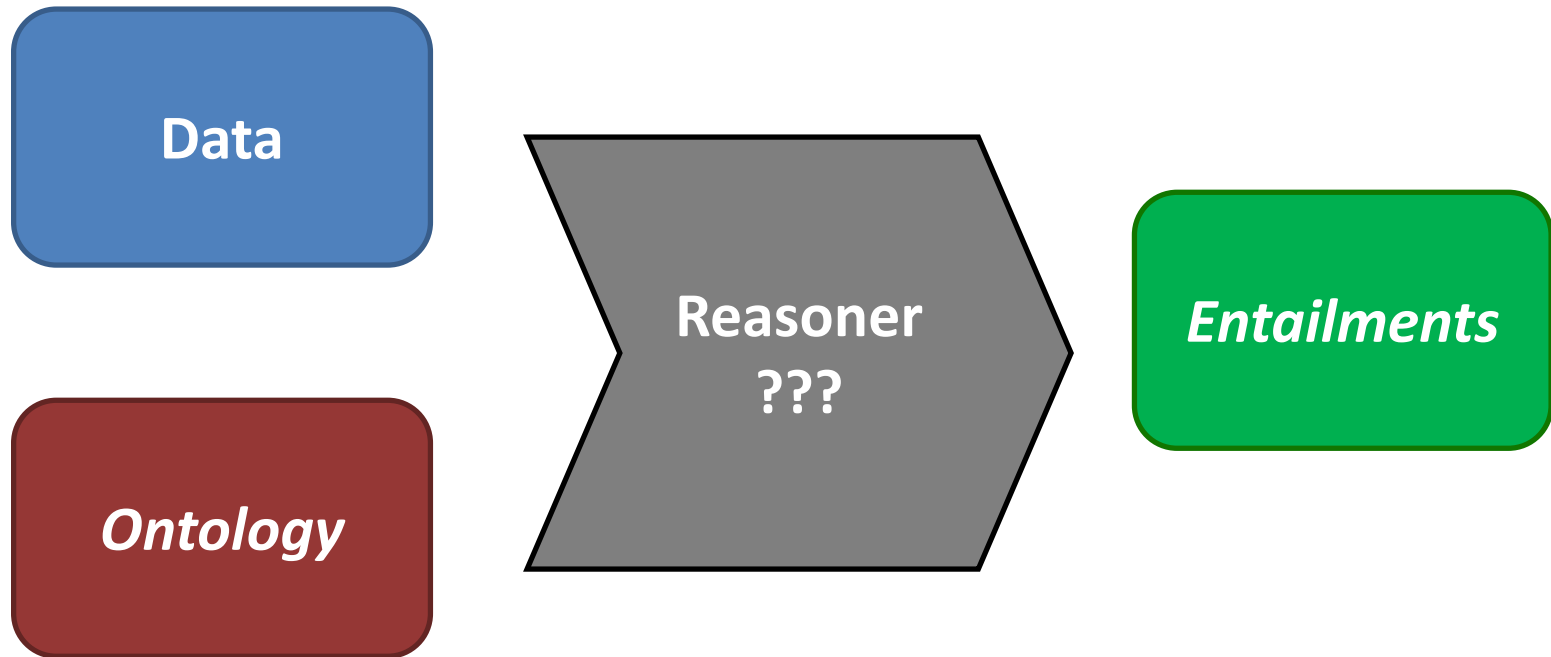
# Options ...

Well great. What are we supposed to do now?

- **Accept incomplete reasoners that halt**
  - You may not get all the entailments ... so what entailments do you get?
- **Accept complete reasoners that may not halt**
  - Java is a language that lets you write programmes that may not halt
- **Restrict OWL so reasoning tasks become decidable**
  - Main problem tackled in Description Logics field: find decidable sublanguages of OWL without turning off too many features (and allowing efficient algorithms)

More next week ...

In the labs ...



- But what is the reasoner *actually* doing? ...

# INCOMPLETE REASONERS THAT HALT

# Incomplete reasoners that halt:

## Works for materialisation

```
:hasDaughter rdfs:subPropertyOf :hasChild .
:hasChild rdf:type owl:AsymmetricProperty ; owl:inverseOf
    :hasParent ;
    rdfs:subPropertyOf :ancestorOf .
:ancestorOf rdf:type owl:TransitiveProperty .
ex:Carmela :hasChild ex:Michael .
ex:Michael :hasDaughter ex:Mary .
ex:Mary a :Person .
:Person owl:equivalentClass
    [ owl:qualifiedCardinality 2 ;
      owl:onProperty :hasParent ;
      owl:onClass :Person ] .
```

```
ex:Michael :hasParent ex:Carmela .
ex:Michael :hasChild ex:Mary .
ex:Carmela :ancestorOf ex:Mary .
```

```
ex:Mary :hasParent _:parent1 . _:parent1 a :Person .
ex:Mary :hasParent _:parent2 . _:parent2 a :Person .
_:parent1 :hasParent _:parent11 . _:parent11 a :Person .
_:parent2 :hasParent _:parent12 . _:parent12 a :Person . ...
```

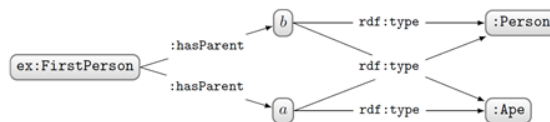
# Incomplete reasoners that halt: for Entailment/Satisfiability Checking?

Ontology Satisfiability:

Does  $\mathcal{O}$  have a “model”?

```
:Person owl:equivalentClass  
[ owl:qualifiedCardinality 2 ; owl:onProperty :hasParent ;  
  owl:onClass :Person ] .  
:FirstPerson a :Person ,  
[ owl:qualifiedCardinality 2 ; owl:onProperty :hasParent ;  
  owl:onClass :Ape ] .
```

So does  $\mathcal{O}$  have a model?



YES! Ontology  $\mathcal{O}$  is Satisfiable!

Entailment checking:

Does  $\mathcal{O}$  entail  $\mathcal{O}'$ ?

```
:hasDaughter rdfs:subPropertyOf :hasChild .  
:hasChild rdf:type owl:AsymmetricProperty ; owl:inverseOf :hasParent ;  
  rdfs:subPropertyOf :ancestorOf .  
:ancestorOf rdf:type owl:TransitiveProperty .  
ex:Carmela :hasChild ex:Michael .  
ex:Michael :hasDaughter ex:Mary .
```

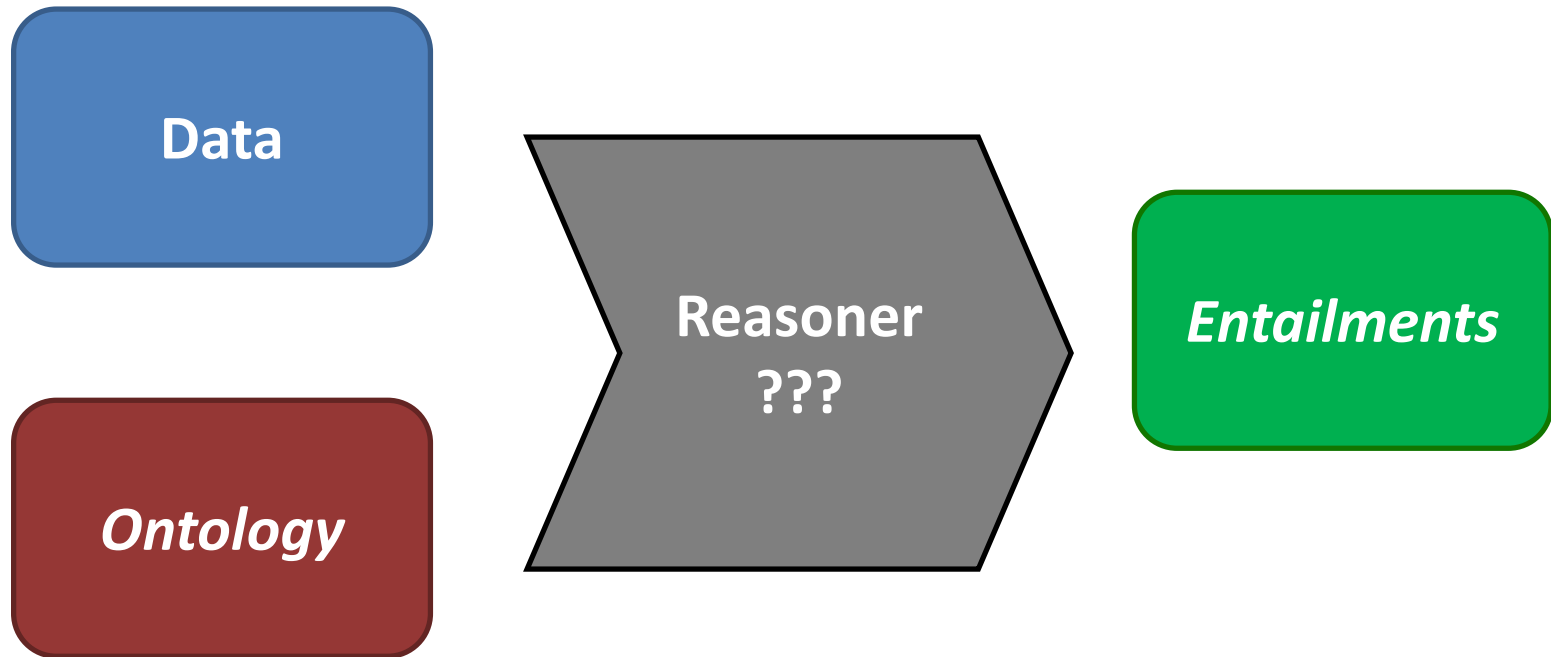
```
ex:Michael :hasParent ex:Carmela .  
ex:Michael :hasChild ex:Mary .  
ex:Carmela :ancestorOf ex:Mary .
```

Alternatively: Are all models of  $\mathcal{O}$  models of  $\mathcal{O}'$  too?

Why can't we have incomplete satisfiability/entailment checkers?

- Both are decision problems (yes/no)
- What would an incomplete answer be? (ye/n)

In the labs ...



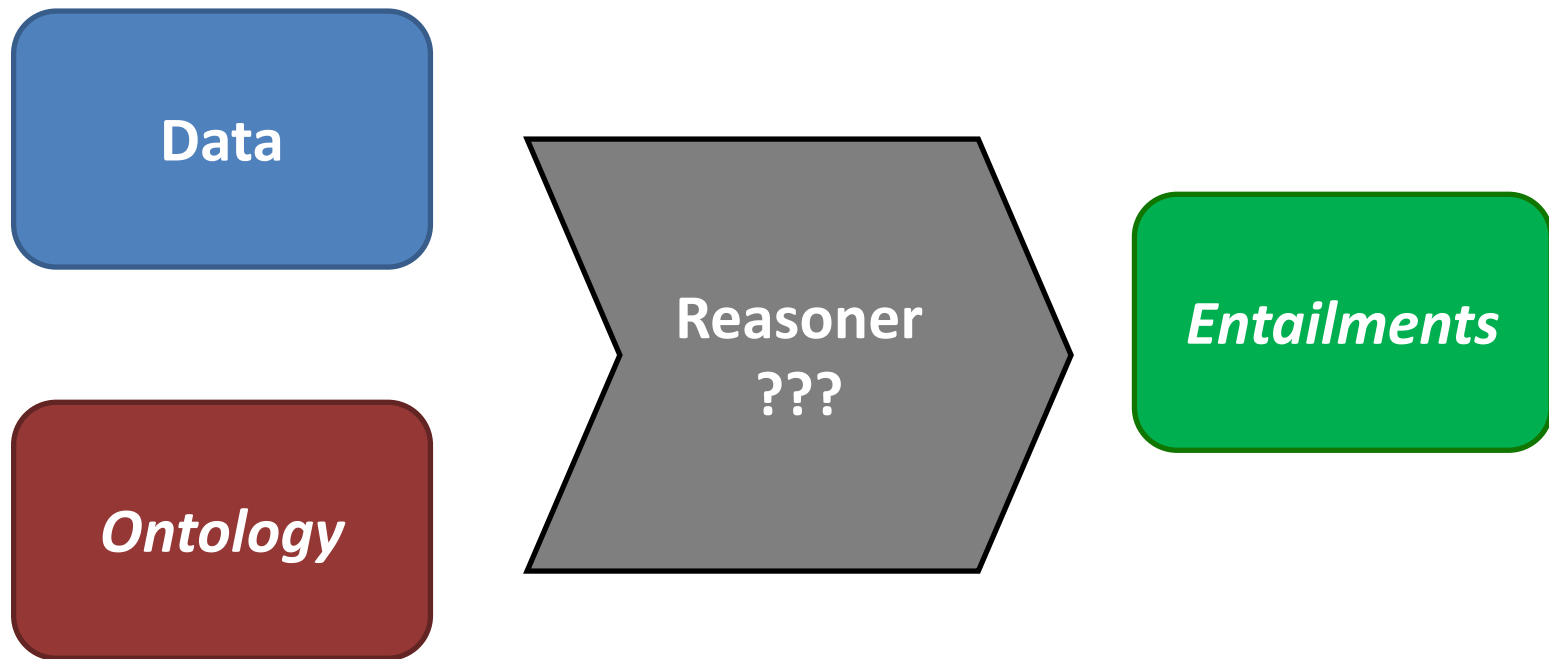
- The reasoner is doing (incomplete) materialisation!

# Recall: RDFS reasoning using “rules”

| ID     | if $G$ matches                                                            | then $G$ $\text{RDFS}_D$ -entails                     |
|--------|---------------------------------------------------------------------------|-------------------------------------------------------|
| rdfD1  | $?x ?p ?l .$ ( $?l$ a literal with datatype IRI $\text{dt}(?l) \in D$ )   | $?x ?p \_ :b . \_ :b \text{ a } \text{dt}(?l) .$      |
| rdfD2  | $?x ?p ?y .$                                                              | $?p \text{ a } \text{rdf:Property} .$                 |
| rdfs1  | $?u \in D$                                                                | $?u \text{ a } \text{rdfs:Datatype} .$                |
| rdfs2  | $?p \text{ rdfs:domain } ?c . ?x ?p ?y .$                                 | $?x \text{ a } ?c .$                                  |
| rdfs3  | $?p \text{ rdfs:range } ?c . ?x ?p ?y .$                                  | $?y \text{ a } ?c .$                                  |
| rdfs4a | $?x ?p ?y .$                                                              | $?x \text{ a } \text{rdfs:Resource} .$                |
| rdfs4b | $?x ?p ?y .$                                                              | $?y \text{ a } \text{rdfs:Resource} .$                |
| rdfs5  | $?p \text{ rdfs:subPropertyOf } ?q . ?x ?p ?y .$                          | $?x ?q ?y .$                                          |
| rdfs6  | $?p \text{ a } \text{rdf:Property} .$                                     | $?p \text{ rdfs:subPropertyOf } ?p .$                 |
| rdfs7  | $?p \text{ rdfs:subPropertyOf } ?q . ?q \text{ rdfs:subPropertyOf } ?r .$ | $?p \text{ rdfs:subPropertyOf } ?r .$                 |
| rdfs8  | $?c \text{ a } \text{rdfs:Class} .$                                       | $?c \text{ rdfs:subClassOf } \text{rdfs:Resource} .$  |
| rdfs9  | $?c \text{ rdfs:subClassOf } ?d . ?x \text{ a } ?c .$                     | $?x \text{ a } ?d .$                                  |
| rdfs10 | $?c \text{ a } \text{rdfs:Class} .$                                       | $?c \text{ rdfs:subClassOf } ?c .$                    |
| rdfs11 | $?c \text{ rdfs:subClassOf } ?d . ?d \text{ rdfs:subClassOf } ?e .$       | $?c \text{ rdfs:subClassOf } ?e .$                    |
| rdfs12 | $?p \text{ a } \text{rdfs:ContainerMembershipProperty} .$                 | $?p \text{ rdfs:subPropertyOf } \text{rdfs:member} .$ |
| rdfs13 | $?d \text{ a } \text{rdfs:Datatype} .$                                    | $?d \text{ rdfs:subClassOf } \text{rdf:Literal} .$    |

(Don't worry about rdfD1, rdfs1, rdfs12, rdfs13)

In the labs ...



- The reasoner is doing (incomplete) materialisation!
  - Using OWL 2 RL/RDF rules that support RDFS *and* OWL (2)

[http://www.w3.org/TR/owl2-profiles/#Reasoning\\_in\\_OWL\\_2\\_RL\\_and\\_RDF\\_Graphs\\_using\\_Rules](http://www.w3.org/TR/owl2-profiles/#Reasoning_in_OWL_2_RL_and_RDF_Graphs_using_Rules)

# Lots of rules ...

- **Goal:** be familiar with idea, not every rule
- Useful for reference
- **Homework:** read over them quickly
  - Let them wash over you 😊



# OWL 2 RL/RDF (rules for OWL)

## Equality

**Table 4. The Semantics of Equality**

|          | If                                                                                                                                                                         | then                                                                                               |                                |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|--------------------------------|
| eq-ref   | $T(?s, ?p, ?o)$                                                                                                                                                            | $T(?s, \text{owl:sameAs}, ?s)$<br>$T(?p, \text{owl:sameAs}, ?p)$<br>$T(?o, \text{owl:sameAs}, ?o)$ |                                |
| eq-sym   | $T(?x, \text{owl:sameAs}, ?y)$                                                                                                                                             | $T(?y, \text{owl:sameAs}, ?x)$                                                                     |                                |
| eq-trans | $T(?x, \text{owl:sameAs}, ?y)$<br>$T(?y, \text{owl:sameAs}, ?z)$                                                                                                           | $T(?x, \text{owl:sameAs}, ?z)$                                                                     |                                |
| eq-rep-s | $T(?s, \text{owl:sameAs}, ?s')$<br>$T(?s, ?p, ?o)$                                                                                                                         | $T(?s', ?p, ?o)$                                                                                   |                                |
| eq-rep-p | $T(?p, \text{owl:sameAs}, ?p')$<br>$T(?s, ?p, ?o)$                                                                                                                         | $T(?s, ?p', ?o)$                                                                                   |                                |
| eq-rep-o | $T(?o, \text{owl:sameAs}, ?o')$<br>$T(?s, ?p, ?o)$                                                                                                                         | $T(?s, ?p, ?o')$                                                                                   |                                |
| eq-diff1 | $T(?x, \text{owl:sameAs}, ?y)$<br>$T(?x, \text{owl:differentFrom}, ?y)$                                                                                                    | false                                                                                              |                                |
| eq-diff2 | $T(?x, \text{rdf:type}, \text{owl:AllDifferent})$<br>$T(?x, \text{owl:members}, ?y)$<br>$\text{LIST}[?y, ?z_1, \dots, ?z_n]$<br>$T(?z_i, \text{owl:sameAs}, ?z_j)$         | false                                                                                              | for each $1 \leq i < j \leq n$ |
| eq-diff3 | $T(?x, \text{rdf:type}, \text{owl:AllDifferent})$<br>$T(?x, \text{owl:distinctMembers}, ?y)$<br>$\text{LIST}[?y, ?z_1, \dots, ?z_n]$<br>$T(?z_i, \text{owl:sameAs}, ?z_j)$ | false                                                                                              | for each $1 \leq i < j \leq n$ |

# OWL 2 RL/RDF (rules for OWL) [Example]

## Property Axioms

What rule(s) could we use for `owl:inverseOf`?

|          |                                                                                                         |                                        |
|----------|---------------------------------------------------------------------------------------------------------|----------------------------------------|
| prp-inv1 | <code>T(?p<sub>1</sub>, owl:inverseOf, ?p<sub>2</sub>)</code><br><code>T(?x, ?p<sub>1</sub>, ?y)</code> | <code>T(?y, ?p<sub>2</sub>, ?x)</code> |
| prp-inv2 | <code>T(?p<sub>1</sub>, owl:inverseOf, ?p<sub>2</sub>)</code><br><code>T(?x, ?p<sub>2</sub>, ?y)</code> | <code>T(?y, ?p<sub>1</sub>, ?x)</code> |

# OWL 2 RL/RDF (rules for OWL)

## Property Axioms

**Table 5. The Semantics of Axioms about Properties**

|           | <b>If</b>                                                                                                | <b>then</b>                                                    |
|-----------|----------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| prp-ap    |                                                                                                          | $T(\text{ap}, \text{rdf:type}, \text{owl:AnnotationProperty})$ |
| prp-dom   | $T(?p, \text{rdfs:domain}, ?c)$<br>$T(?x, ?p, ?y)$                                                       | $T(?x, \text{rdf:type}, ?c)$                                   |
| prp-rng   | $T(?p, \text{rdfs:range}, ?c)$<br>$T(?x, ?p, ?y)$                                                        | $T(?y, \text{rdf:type}, ?c)$                                   |
| prp-fp    | $T(?p, \text{rdf:type}, \text{owl:FunctionalProperty})$<br>$T(?x, ?p, ?y_1)$<br>$T(?x, ?p, ?y_2)$        | $T(?y_1, \text{owl:sameAs}, ?y_2)$                             |
| prp-ifp   | $T(?p, \text{rdf:type}, \text{owl:InverseFunctionalProperty})$<br>$T(?x_1, ?p, ?y)$<br>$T(?x_2, ?p, ?y)$ | $T(?x_1, \text{owl:sameAs}, ?x_2)$                             |
| prp-irp   | $T(?p, \text{rdf:type}, \text{owl:IrreflexiveProperty})$<br>$T(?x, ?p, ?x)$                              | false                                                          |
| prp-symp  | $T(?p, \text{rdf:type}, \text{owl:SymmetricProperty})$<br>$T(?x, ?p, ?y)$                                | $T(?y, ?p, ?x)$                                                |
| prp-asymp | $T(?p, \text{rdf:type}, \text{owl:AsymmetricProperty})$<br>$T(?x, ?p, ?y)$<br>$T(?y, ?p, ?x)$            | false                                                          |
| prp-trp   | $T(?p, \text{rdf:type}, \text{owl:TransitiveProperty})$<br>$T(?x, ?p, ?y)$<br>$T(?y, ?p, ?z)$            | $T(?x, ?p, ?z)$                                                |

# OWL 2 RL/RDF (rules for OWL)

## Property Axioms

|          |                                                                                                                                                                                 |                         |                                |
|----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------|--------------------------------|
| prp-spo1 | $T(?p_1, \text{rdfs:subPropertyOf}, ?p_2)$<br>$T(?x, ?p_1, ?y)$                                                                                                                 | $T(?x, ?p_2, ?y)$       |                                |
| prp-spo2 | $T(?p, \text{owl:propertyChainAxiom}, ?x)$<br>$\text{LIST}[?x, ?p_1, \dots, ?p_n]$<br>$T(?u_1, ?p_1, ?u_2)$<br>$T(?u_2, ?p_2, ?u_3)$<br>$\dots$<br>$T(?u_n, ?p_n, ?u_{n+1})$    | $T(?u_1, ?p, ?u_{n+1})$ |                                |
| prp-eqp1 | $T(?p_1, \text{owl:equivalentProperty}, ?p_2)$<br>$T(?x, ?p_1, ?y)$                                                                                                             | $T(?x, ?p_2, ?y)$       |                                |
| prp-eqp2 | $T(?p_1, \text{owl:equivalentProperty}, ?p_2)$<br>$T(?x, ?p_2, ?y)$                                                                                                             | $T(?x, ?p_1, ?y)$       |                                |
| prp-pdw  | $T(?p_1, \text{owl:propertyDisjointWith}, ?p_2)$<br>$T(?x, ?p_1, ?y)$<br>$T(?x, ?p_2, ?y)$                                                                                      | false                   |                                |
| prp-adp  | $T(?x, \text{rdf:type}, \text{owl:AllDisjointProperties})$<br>$T(?x, \text{owl:members}, ?y)$<br>$\text{LIST}[?y, ?p_1, \dots, ?p_n]$<br>$T(?u, ?p_1, ?v)$<br>$T(?u, ?p_j, ?v)$ | false                   | for each $1 \leq i < j \leq n$ |
| prp-inv1 | $T(?p_1, \text{owl:inverseOf}, ?p_2)$<br>$T(?x, ?p_1, ?y)$                                                                                                                      | $T(?y, ?p_2, ?x)$       |                                |
| prp-inv2 | $T(?p_1, \text{owl:inverseOf}, ?p_2)$<br>$T(?x, ?p_2, ?y)$                                                                                                                      | $T(?y, ?p_1, ?x)$       |                                |

# OWL 2 RL/RDF (rules for OWL)

## Property Axioms

|          |                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                    |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------|
| prp-key  | <code>T(?c, owl:hasKey, ?u)</code><br><code>LIST[?u, ?p<sub>1</sub>, ..., ?p<sub>n</sub>]</code><br><code>T(?x, rdf:type, ?c)</code><br><code>T(?x, ?p<sub>1</sub>, ?z<sub>1</sub>)</code><br><code>...</code><br><code>T(?x, ?p<sub>n</sub>, ?z<sub>n</sub>)</code><br><code>T(?y, rdf:type, ?c)</code><br><code>T(?y, ?p<sub>1</sub>, ?z<sub>1</sub>)</code><br><code>...</code><br><code>T(?y, ?p<sub>n</sub>, ?z<sub>n</sub>)</code> | <code>T(?x, owl:sameAs, ?y)</code> |
| prp-npa1 | <code>T(?x, owl:sourceIndividual, ?i<sub>1</sub>)</code><br><code>T(?x, owl:assertionProperty, ?p)</code><br><code>T(?x, owl:targetIndividual, ?i<sub>2</sub>)</code><br><code>T(?i<sub>1</sub>, ?p, ?i<sub>2</sub>)</code>                                                                                                                                                                                                              | <code>false</code>                 |
| prp-npa2 | <code>T(?x, owl:sourceIndividual, ?i)</code><br><code>T(?x, owl:assertionProperty, ?p)</code><br><code>T(?x, owl:targetValue, ?lt)</code><br><code>T(?i, ?p, ?lt)</code>                                                                                                                                                                                                                                                                 | <code>false</code>                 |

# OWL 2 RL/RDF (rules for OWL) [Example]

## Class Axioms

What rule(s) could we use for `owl:disjointWith`?

|                     |                                                                                                                                                                  |                    |
|---------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------|
| <code>cax-dw</code> | <code>T(?c<sub>1</sub>, owl:disjointWith, ?c<sub>2</sub>)</code><br><code>T(?x, rdf:type, ?c<sub>1</sub>)</code><br><code>T(?x, rdf:type, ?c<sub>2</sub>)</code> | <code>false</code> |
|---------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------|

# OWL 2 RL/RDF (rules for OWL)

## Class Axioms

**Table 7. The Semantics of Class Axioms**

|          | <b>If</b>                                                                                                                                                                                              | <b>then</b>                    |                                |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------|--------------------------------|
| cax-sco  | $T(?c_1, \text{rdfs:subClassOf}, ?c_2)$<br>$T(?x, \text{rdf:type}, ?c_1)$                                                                                                                              | $T(?x, \text{rdf:type}, ?c_2)$ |                                |
| cax-eqc1 | $T(?c_1, \text{owl:equivalentClass}, ?c_2)$<br>$T(?x, \text{rdf:type}, ?c_1)$                                                                                                                          | $T(?x, \text{rdf:type}, ?c_2)$ |                                |
| cax-eqc2 | $T(?c_1, \text{owl:equivalentClass}, ?c_2)$<br>$T(?x, \text{rdf:type}, ?c_2)$                                                                                                                          | $T(?x, \text{rdf:type}, ?c_1)$ |                                |
| cax-dw   | $T(?c_1, \text{owl:disjointWith}, ?c_2)$<br>$T(?x, \text{rdf:type}, ?c_1)$<br>$T(?x, \text{rdf:type}, ?c_2)$                                                                                           | false                          |                                |
| cax-adc  | $T(?x, \text{rdf:type}, \text{owl:AllDisjointClasses})$<br>$T(?x, \text{owl:members}, ?y)$<br>$\text{LIST}[?y, ?c_1, \dots, ?c_n]$<br>$T(?z, \text{rdf:type}, ?c_1)$<br>$T(?z, \text{rdf:type}, ?c_j)$ | false                          | for each $1 \leq i < j \leq n$ |

# OWL 2 RL/RDF (rules for OWL) [Example]

## Class definitions

What rule(s) could we use for `owl:intersectionOf`?

|          |                                                                                                                                                                                                                                                                 |                                                                                                                                                     |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------|
| cls-int1 | <code>T(?c, owl:intersectionOf, ?x)</code><br><code>LIST[?x, ?c<sub>1</sub>, ..., ?c<sub>n</sub>]</code><br><code>T(?y, rdf:type, ?c<sub>1</sub>)</code><br><code>T(?y, rdf:type, ?c<sub>2</sub>)</code><br>...<br><code>T(?y, rdf:type, ?c<sub>n</sub>)</code> | <code>T(?y, rdf:type, ?c)</code>                                                                                                                    |
| cls-int2 | <code>T(?c, owl:intersectionOf, ?x)</code><br><code>LIST[?x, ?c<sub>1</sub>, ..., ?c<sub>n</sub>]</code><br><code>T(?y, rdf:type, ?c)</code>                                                                                                                    | <code>T(?y, rdf:type, ?c<sub>1</sub>)</code><br><code>T(?y, rdf:type, ?c<sub>2</sub>)</code><br>...<br><code>T(?y, rdf:type, ?c<sub>n</sub>)</code> |

What rule(s) could we use for `owl:allValuesFrom`?

|         |                                                                                                                                                       |                                  |
|---------|-------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------|
| cls-avf | <code>T(?x, owl:allValuesFrom, ?y)</code><br><code>T(?x, owl:onProperty, ?p)</code><br><code>T(?u, rdf:type, ?x)</code><br><code>T(?u, ?p, ?v)</code> | <code>T(?v, rdf:type, ?y)</code> |
|---------|-------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------|

No way to write rule for “opposite” direction:

if ... `T(?x, owl:allValuesFrom, ?y)` and `T(?x, owl:onProperty, ?p)`  
and where `T(?u, ?p, ?v)` implies `?v` of `rdf:type ?y` then `T(?u, rdf:type, ?x)`

# OWL 2 RL/RDF (rules for OWL)

## Class definitions

**Table 6. The Semantics of Classes**

|              | If                                                                                                                                                                                              | then                                                                                                          |                            |
|--------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------|----------------------------|
| cls-thing    |                                                                                                                                                                                                 | $T(\text{owl:Thing}, \text{rdf:type}, \text{owl:Class})$                                                      |                            |
| cls-nothing1 |                                                                                                                                                                                                 | $T(\text{owl:Nothing}, \text{rdf:type}, \text{owl:Class})$                                                    |                            |
| cls-nothing2 | $T(?x, \text{rdf:type}, \text{owl:Nothing})$                                                                                                                                                    | false                                                                                                         |                            |
| cls-int1     | $T(?c, \text{owl:intersectionOf}, ?x)$<br>$\text{LIST}[?x, ?c_1, \dots, ?c_n]$<br>$T(?y, \text{rdf:type}, ?c_1)$<br>$T(?y, \text{rdf:type}, ?c_2)$<br>$\dots$<br>$T(?y, \text{rdf:type}, ?c_n)$ | $T(?y, \text{rdf:type}, ?c)$                                                                                  |                            |
| cls-int2     | $T(?c, \text{owl:intersectionOf}, ?x)$<br>$\text{LIST}[?x, ?c_1, \dots, ?c_n]$<br>$T(?y, \text{rdf:type}, ?c)$                                                                                  | $T(?y, \text{rdf:type}, ?c_1)$<br>$T(?y, \text{rdf:type}, ?c_2)$<br>$\dots$<br>$T(?y, \text{rdf:type}, ?c_n)$ |                            |
| cls-uni      | $T(?c, \text{owl:unionOf}, ?x)$<br>$\text{LIST}[?x, ?c_1, \dots, ?c_n]$<br>$T(?y, \text{rdf:type}, ?c_1)$                                                                                       | $T(?y, \text{rdf:type}, ?c)$                                                                                  | for each $1 \leq i \leq n$ |
| cls-com      | $T(?c_1, \text{owl:complementOf}, ?c_2)$<br>$T(?x, \text{rdf:type}, ?c_1)$<br>$T(?x, \text{rdf:type}, ?c_2)$                                                                                    | false                                                                                                         |                            |

# OWL 2 RL/RDF (rules for OWL)

## Class definitions

|           |                                                                                                                                                                                                                                       |                                                            |
|-----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------|
| cls-svf1  | <code>T(?x, owl:someValuesFrom, ?y)</code><br><code>T(?x, owl:onProperty, ?p)</code><br><code>T(?u, ?p, ?v)</code><br><code>T(?v, rdf:type, ?y)</code>                                                                                | <code>T(?u, rdf:type, ?x)</code>                           |
| cls-svf2  | <code>T(?x, owl:someValuesFrom, owl:Thing)</code><br><code>T(?x, owl:onProperty, ?p)</code><br><code>T(?u, ?p, ?v)</code>                                                                                                             | <code>T(?u, rdf:type, ?x)</code>                           |
| cls-avf   | <code>T(?x, owl:allValuesFrom, ?y)</code><br><code>T(?x, owl:onProperty, ?p)</code><br><code>T(?u, rdf:type, ?x)</code><br><code>T(?u, ?p, ?v)</code>                                                                                 | <code>T(?v, rdf:type, ?y)</code>                           |
| cls-hv1   | <code>T(?x, owl:hasValue, ?y)</code><br><code>T(?x, owl:onProperty, ?p)</code><br><code>T(?u, rdf:type, ?x)</code>                                                                                                                    | <code>T(?u, ?p, ?y)</code>                                 |
| cls-hv2   | <code>T(?x, owl:hasValue, ?y)</code><br><code>T(?x, owl:onProperty, ?p)</code><br><code>T(?u, ?p, ?y)</code>                                                                                                                          | <code>T(?u, rdf:type, ?x)</code>                           |
| cls-maxc1 | <code>T(?x, owl:maxCardinality, "0"^^xsd:nonNegativeInteger)</code><br><code>T(?x, owl:onProperty, ?p)</code><br><code>T(?u, rdf:type, ?x)</code><br><code>T(?u, ?p, ?y)</code>                                                       | false                                                      |
| cls-maxc2 | <code>T(?x, owl:maxCardinality, "1"^^xsd:nonNegativeInteger)</code><br><code>T(?x, owl:onProperty, ?p)</code><br><code>T(?u, rdf:type, ?x)</code><br><code>T(?u, ?p, ?y<sub>1</sub>)</code><br><code>T(?u, ?p, ?y<sub>2</sub>)</code> | <code>T(?y<sub>1</sub>, owl:sameAs, ?y<sub>2</sub>)</code> |

# OWL 2 RL/RDF (rules for OWL)

## Class definitions

|            |                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                     |
|------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------|
| cls-maxqc1 | <code>T(?x, owl:maxQualifiedCardinality, "0"^^xsd:nonNegativeInteger)</code><br><code>T(?x, owl:onProperty, ?p)</code><br><code>T(?x, owl:onClass, ?c)</code><br><code>T(?u, rdf:type, ?x)</code><br><code>T(?u, ?p, ?y)</code><br><code>T(?y, rdf:type, ?c)</code>                                                                                                                   | false                                                                                               |
| cls-maxqc2 | <code>T(?x, owl:maxQualifiedCardinality, "0"^^xsd:nonNegativeInteger)</code><br><code>T(?x, owl:onProperty, ?p)</code><br><code>T(?x, owl:onClass, owl:Thing)</code><br><code>T(?u, rdf:type, ?x)</code><br><code>T(?u, ?p, ?y)</code>                                                                                                                                                | false                                                                                               |
| cls-maxqc3 | <code>T(?x, owl:maxQualifiedCardinality, "1"^^xsd:nonNegativeInteger)</code><br><code>T(?x, owl:onProperty, ?p)</code><br><code>T(?x, owl:onClass, ?c)</code><br><code>T(?u, rdf:type, ?x)</code><br><code>T(?u, ?p, ?y<sub>1</sub>)</code><br><code>T(?y<sub>1</sub>, rdf:type, ?c)</code><br><code>T(?u, ?p, ?y<sub>2</sub>)</code><br><code>T(?y<sub>2</sub>, rdf:type, ?c)</code> | <code>T(?y<sub>1</sub>, owl:sameAs, ?y<sub>2</sub>)</code>                                          |
| cls-maxqc4 | <code>T(?x, owl:maxQualifiedCardinality, "1"^^xsd:nonNegativeInteger)</code><br><code>T(?x, owl:onProperty, ?p)</code><br><code>T(?x, owl:onClass, owl:Thing)</code><br><code>T(?u, rdf:type, ?x)</code><br><code>T(?u, ?p, ?y<sub>1</sub>)</code><br><code>T(?u, ?p, ?y<sub>2</sub>)</code>                                                                                          | <code>T(?y<sub>1</sub>, owl:sameAs, ?y<sub>2</sub>)</code>                                          |
| cls-oo     | <code>T(?c, owl:oneOf, ?x)</code><br><code>LIST[?x, ?y<sub>1</sub>, ..., ?y<sub>n</sub>]</code>                                                                                                                                                                                                                                                                                       | <code>T(?y<sub>1</sub>, rdf:type, ?c)</code><br>...<br><code>T(?y<sub>n</sub>, rdf:type, ?c)</code> |

# OWL 2 RL/RDF (rules for OWL) [Example]

## Schema

|         |                                                                                                                 |                                              |
|---------|-----------------------------------------------------------------------------------------------------------------|----------------------------------------------|
| cax-sco | <code>T(?c<sub>1</sub>, rdfs:subClassOf, ?c<sub>2</sub>)</code><br><code>T(?x, rdf:type, ?c<sub>1</sub>)</code> | <code>T(?x, rdf:type, ?c<sub>2</sub>)</code> |
|---------|-----------------------------------------------------------------------------------------------------------------|----------------------------------------------|

... but what other rule(s) are we missing for `rdfs:subClassOf`?

|          |                                                                                                                                    |                                                                     |
|----------|------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------|
| scm-sco  | <code>T(?c<sub>1</sub>, rdfs:subClassOf, ?c<sub>2</sub>)</code><br><code>T(?c<sub>2</sub>, rdfs:subClassOf, ?c<sub>3</sub>)</code> | <code>T(?c<sub>1</sub>, rdfs:subClassOf, ?c<sub>3</sub>)</code>     |
| scm-eqc2 | <code>T(?c<sub>1</sub>, rdfs:subClassOf, ?c<sub>2</sub>)</code><br><code>T(?c<sub>2</sub>, rdfs:subClassOf, ?c<sub>1</sub>)</code> | <code>T(?c<sub>1</sub>, owl:equivalentClass, ?c<sub>2</sub>)</code> |

...

# OWL 2 RL/RDF (rules for OWL)

## Schema

**Table 9. The Semantics of Schema Vocabulary**

|          | If                                                                                       | then                                                                                                                                                                                       |
|----------|------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| scm-cls  | $T(?c, \text{rdf:type}, \text{owl:Class})$                                               | $T(?c, \text{rdfs:subClassOf}, ?c)$<br>$T(?c, \text{owl:equivalentClass}, ?c)$<br>$T(?c, \text{rdfs:subClassOf}, \text{owl:Thing})$<br>$T(\text{owl:Nothing}, \text{rdfs:subClassOf}, ?c)$ |
| scm-sco  | $T(?c_1, \text{rdfs:subClassOf}, ?c_2)$<br>$T(?c_2, \text{rdfs:subClassOf}, ?c_3)$       | $T(?c_1, \text{rdfs:subClassOf}, ?c_3)$                                                                                                                                                    |
| scm-eqc1 | $T(?c_1, \text{owl:equivalentClass}, ?c_2)$                                              | $T(?c_1, \text{rdfs:subClassOf}, ?c_2)$<br>$T(?c_2, \text{rdfs:subClassOf}, ?c_1)$                                                                                                         |
| scm-eqc2 | $T(?c_1, \text{rdfs:subClassOf}, ?c_2)$<br>$T(?c_2, \text{rdfs:subClassOf}, ?c_1)$       | $T(?c_1, \text{owl:equivalentClass}, ?c_2)$                                                                                                                                                |
| scm-op   | $T(?p, \text{rdf:type}, \text{owl:ObjectProperty})$                                      | $T(?p, \text{rdfs:subPropertyOf}, ?p)$<br>$T(?p, \text{owl:equivalentProperty}, ?p)$                                                                                                       |
| scm-dp   | $T(?p, \text{rdf:type}, \text{owl:DatatypeProperty})$                                    | $T(?p, \text{rdfs:subPropertyOf}, ?p)$<br>$T(?p, \text{owl:equivalentProperty}, ?p)$                                                                                                       |
| scm-spo  | $T(?p_1, \text{rdfs:subPropertyOf}, ?p_2)$<br>$T(?p_2, \text{rdfs:subPropertyOf}, ?p_3)$ | $T(?p_1, \text{rdfs:subPropertyOf}, ?p_3)$                                                                                                                                                 |
| scm-ep1  | $T(?p_1, \text{owl:equivalentProperty}, ?p_2)$                                           | $T(?p_1, \text{rdfs:subPropertyOf}, ?p_2)$<br>$T(?p_2, \text{rdfs:subPropertyOf}, ?p_1)$                                                                                                   |
| scm-ep2  | $T(?p_1, \text{rdfs:subPropertyOf}, ?p_2)$<br>$T(?p_2, \text{rdfs:subPropertyOf}, ?p_1)$ | $T(?p_1, \text{owl:equivalentProperty}, ?p_2)$                                                                                                                                             |

# OWL 2 RL/RDF (rules for OWL)

## Schema

|          |                                                                                                                                                                                                                                    |                                           |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------|
| scm-dom2 | <code>T(?p2, rdfs:domain, ?c)</code><br><code>T(?p1, rdfs:subPropertyOf, ?p2)</code>                                                                                                                                               | <code>T(?p1, rdfs:domain, ?c)</code>      |
| scm-rng1 | <code>T(?p, rdfs:range, ?c1)</code><br><code>T(?c1, rdfs:subClassOf, ?c2)</code>                                                                                                                                                   | <code>T(?p, rdfs:range, ?c2)</code>       |
| scm-rng2 | <code>T(?p2, rdfs:range, ?c)</code><br><code>T(?p1, rdfs:subPropertyOf, ?p2)</code>                                                                                                                                                | <code>T(?p1, rdfs:range, ?c)</code>       |
| scm-hv   | <code>T(?c1, owl:hasValue, ?i)</code><br><code>T(?c1, owl:onProperty, ?p1)</code><br><code>T(?c2, owl:hasValue, ?i)</code><br><code>T(?c2, owl:onProperty, ?p2)</code><br><code>T(?p1, rdfs:subPropertyOf, ?p2)</code>             | <code>T(?c1, rdfs:subClassOf, ?c2)</code> |
| scm-svf1 | <code>T(?c1, owl:someValuesFrom, ?y1)</code><br><code>T(?c1, owl:onProperty, ?p)</code><br><code>T(?c2, owl:someValuesFrom, ?y2)</code><br><code>T(?c2, owl:onProperty, ?p)</code><br><code>T(?y1, rdfs:subClassOf, ?y2)</code>    | <code>T(?c1, rdfs:subClassOf, ?c2)</code> |
| scm-svf2 | <code>T(?c1, owl:someValuesFrom, ?y)</code><br><code>T(?c1, owl:onProperty, ?p1)</code><br><code>T(?c2, owl:someValuesFrom, ?y)</code><br><code>T(?c2, owl:onProperty, ?p2)</code><br><code>T(?p1, rdfs:subPropertyOf, ?p2)</code> | <code>T(?c1, rdfs:subClassOf, ?c2)</code> |

# OWL 2 RL/RDF (rules for OWL)

## Schema

|          |                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                          |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| scm-avf1 | <code>T(?c<sub>1</sub>, owl:allValuesFrom, ?y<sub>1</sub>)</code><br><code>T(?c<sub>1</sub>, owl:onProperty, ?p)</code><br><code>T(?c<sub>2</sub>, owl:allValuesFrom, ?y<sub>2</sub>)</code><br><code>T(?c<sub>2</sub>, owl:onProperty, ?p)</code><br><code>T(?y<sub>1</sub>, rdfs:subClassOf, ?y<sub>2</sub>)</code>    | <code>T(?c<sub>1</sub>, rdfs:subClassOf, ?c<sub>2</sub>)</code>                                                                                                          |
| scm-avf2 | <code>T(?c<sub>1</sub>, owl:allValuesFrom, ?y)</code><br><code>T(?c<sub>1</sub>, owl:onProperty, ?p<sub>1</sub>)</code><br><code>T(?c<sub>2</sub>, owl:allValuesFrom, ?y)</code><br><code>T(?c<sub>2</sub>, owl:onProperty, ?p<sub>2</sub>)</code><br><code>T(?p<sub>1</sub>, rdfs:subPropertyOf, ?p<sub>2</sub>)</code> | <code>T(?c<sub>2</sub>, rdfs:subClassOf, ?c<sub>1</sub>)</code>                                                                                                          |
| scm-int  | <code>T(?c, owl:intersectionOf, ?x)</code><br><code>LIST[?x, ?c<sub>1</sub>, ..., ?c<sub>n</sub>]</code>                                                                                                                                                                                                                 | <code>T(?c, rdfs:subClassOf, ?c<sub>1</sub>)</code><br><code>T(?c, rdfs:subClassOf, ?c<sub>2</sub>)</code><br>...<br><code>T(?c, rdfs:subClassOf, ?c<sub>n</sub>)</code> |
| scm-uni  | <code>T(?c, owl:unionOf, ?x)</code><br><code>LIST[?x, ?c<sub>1</sub>, ..., ?c<sub>n</sub>]</code>                                                                                                                                                                                                                        | <code>T(?c<sub>1</sub>, rdfs:subClassOf, ?c)</code><br><code>T(?c<sub>2</sub>, rdfs:subClassOf, ?c)</code><br>...<br><code>T(?c<sub>n</sub>, rdfs:subClassOf, ?c)</code> |

# OWL 2 RL/RDF (rules for OWL)

## Datatypes

Want to capture:

```
"2"^^xsd:integer owl:sameAs "2.0"^^xsd:decimal .
```

```
"2"^^xsd:integer owl:differentFrom "3.0"^^xsd:decimal .
```

```
"2"^^xsd:integer owl:differentFrom "2.0"^^xsd:string .
```

(Literals allowed in subject positions while reasoning, removed afterwards)

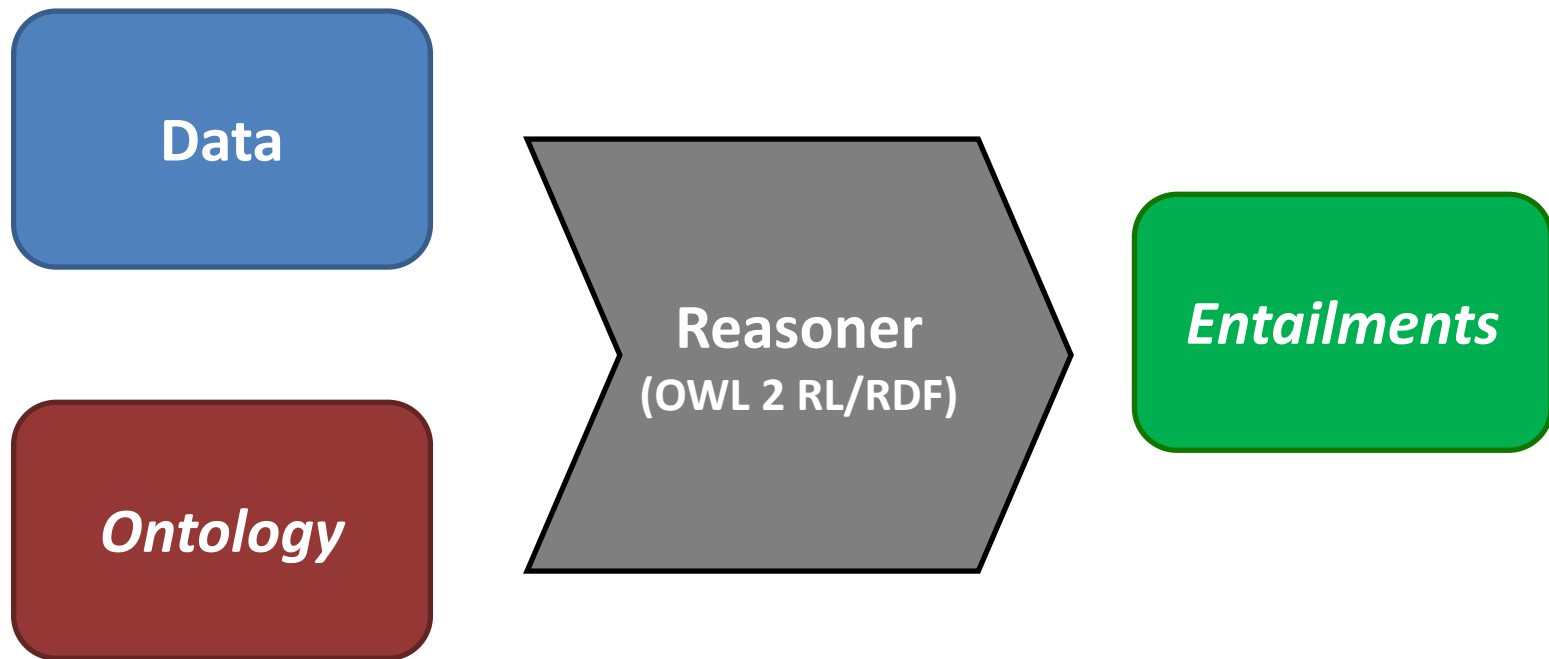
# OWL 2 RL/RDF (rules for OWL)

## Datatypes

**Table 8.** The Semantics of Datatypes

|             | If                           | then                                           |                                                                                                                                                 |
|-------------|------------------------------|------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| dt-type1    |                              | $T(dt, \text{rdf:type}, \text{rdfs:Datatype})$ | for each datatype $dt$ supported in OWL 2 RL                                                                                                    |
| dt-type2    |                              | $T(lt, \text{rdf:type}, dt)$                   | for each literal $lt$ and each datatype $dt$ supported in OWL 2 RL such that the data value of $lt$ is contained in the value space of $dt$     |
| dt-eq       |                              | $T(lt_1, \text{owl:sameAs}, lt_2)$             | for all literals $lt_1$ and $lt_2$ with the same data value                                                                                     |
| dt-diff     |                              | $T(lt_1, \text{owl:differentFrom}, lt_2)$      | for all literals $lt_1$ and $lt_2$ with different data values                                                                                   |
| dt-not-type | $T(lt, \text{rdf:type}, dt)$ | false                                          | for each literal $lt$ and each datatype $dt$ supported in OWL 2 RL such that the data value of $lt$ is not contained in the value space of $dt$ |

# In the labs ...



- Applies these OWL 2 RL/RDF rules recursively until nothing new is found

[http://www.w3.org/TR/owl2-profiles/#Reasoning in OWL 2 RL and RDF Graphs using Rules](http://www.w3.org/TR/owl2-profiles/#Reasoning_in_OWL_2_RL_and_RDF_Graphs_using_Rules)

# Why Incomplete?

## Missing Features

owl:ReflexiveProperty



```
:similarTo rdf:type owl:ReflexiveProperty .
```

```
⇒ ex:Connie :similarTo ex:Connie .
```

```
ex:Freddie :similarTo ex:Freddie .
```

(everything :similarTo itself)

# Why Incomplete?

## Missing Features

owl:hasSelf (Self) [i]



Self(*:loves*)

rdf:type



ex:Vincent

*:loves*

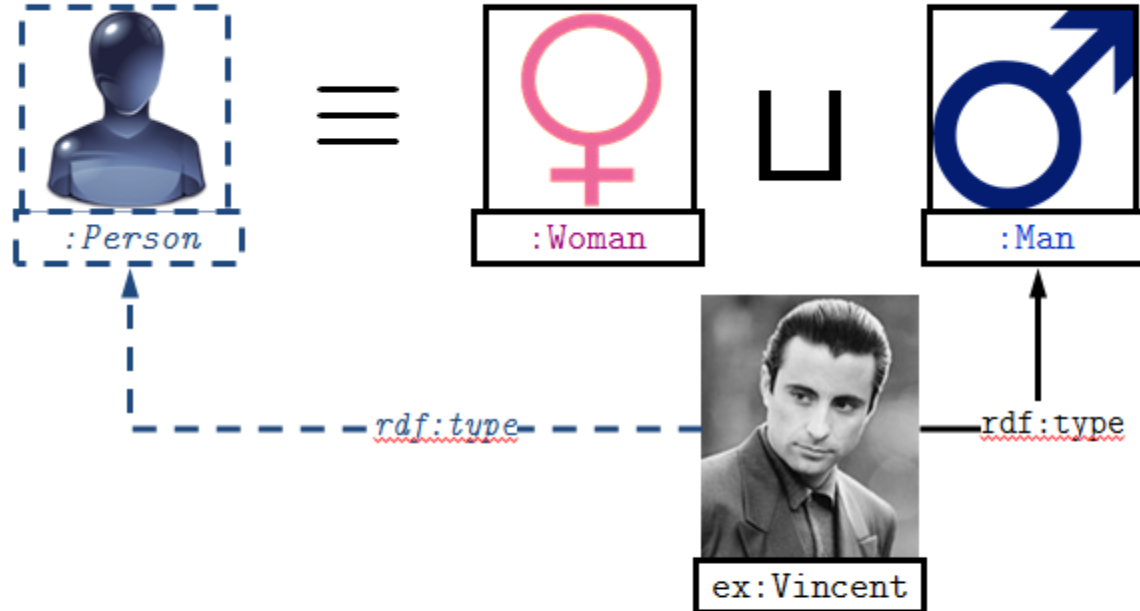
```
ex:Vincent rdf:type :Narcissist .  
:Narcissist rdfs:subClassOf  
  [ owl:hasSelf true ; owl:onProperty :loves ]  
⇒ ex:Vincent :loves ex:Vincent .
```

# Why Incomplete?

## Incomplete for some Features

|         |                                                                                                           |                              |                            |
|---------|-----------------------------------------------------------------------------------------------------------|------------------------------|----------------------------|
| cls-uni | $T(?c, \text{owl:unionOf}, ?x)$<br>$\text{LIST}[?x, ?c_1, \dots, ?c_n]$<br>$T(?y, \text{rdf:type}, ?c_i)$ | $T(?y, \text{rdf:type}, ?c)$ | for each $1 \leq i \leq n$ |
|---------|-----------------------------------------------------------------------------------------------------------|------------------------------|----------------------------|

owl:unionOf ( $\sqcup$ )



ex:Vincent rdf:type :Man .

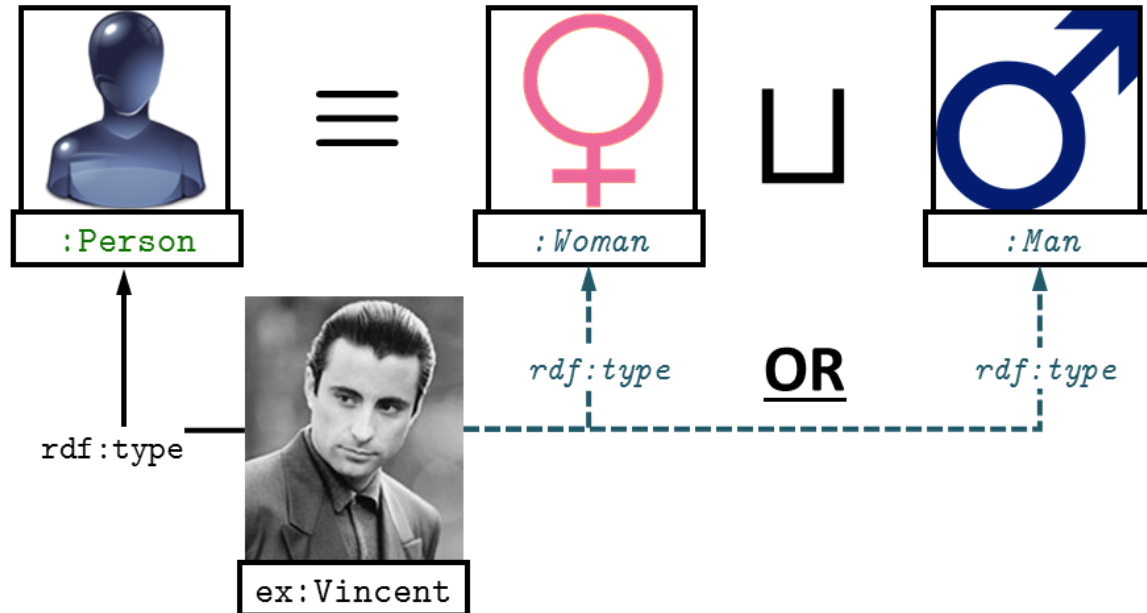
:Person owl:equivalentClass [ owl:unionOf ( :Woman :Man ) ]

$\Rightarrow$  ex:Vincent rdf:type :Person .

# Why Incomplete?

Incomplete for some Features

owl:unionOf (⊔) [ii]



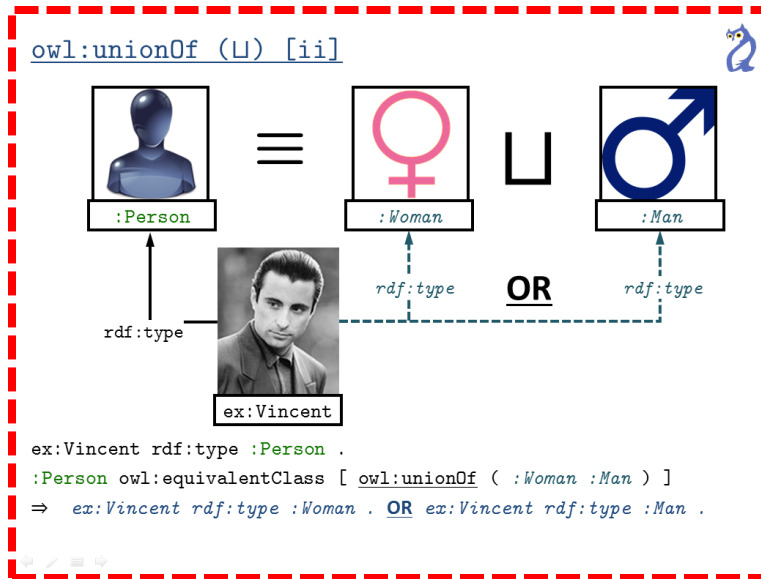
```
ex:Vincent rdf:type :Person .
```

```
:Person owl:equivalentClass [ owl:unionOf ( :Woman :Man ) ]
```

```
⇒ ex:Vincent rdf:type :Woman . OR ex:Vincent rdf:type :Man .
```

# Why Incomplete?

## Incomplete for some Features



`ex:Vincent rdf:type :Person , :Godfather .`

`:Godfather owl:disjointWith :Woman .`

`:Person owl:equivalentClass [ owl:unionOf ( :Woman :Man ) ]`

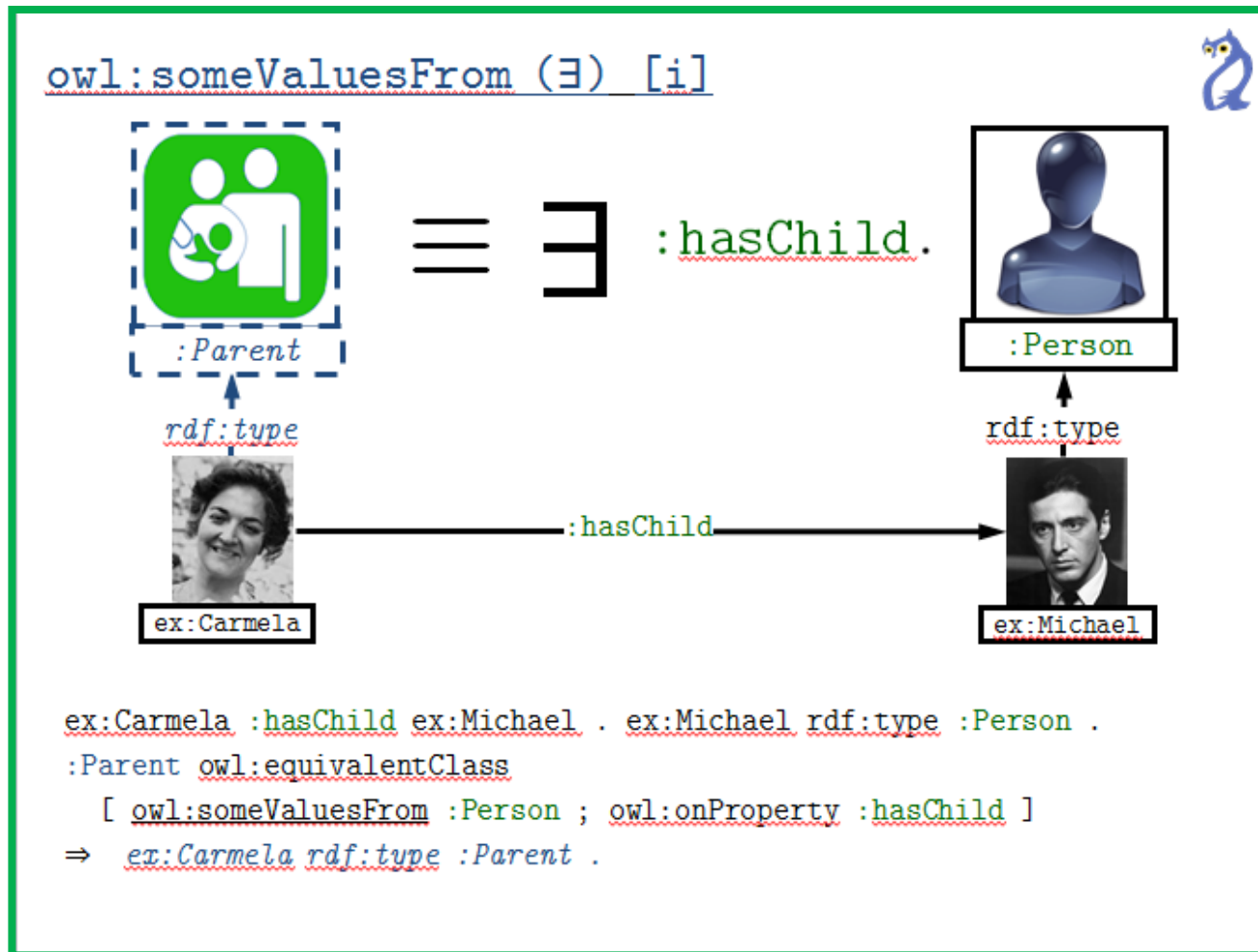
`⇒ ex:Vincent rdf:type :Man .`

Will not get this (valid) entailment with OWL 2 RL/RDF

# Why Incomplete?

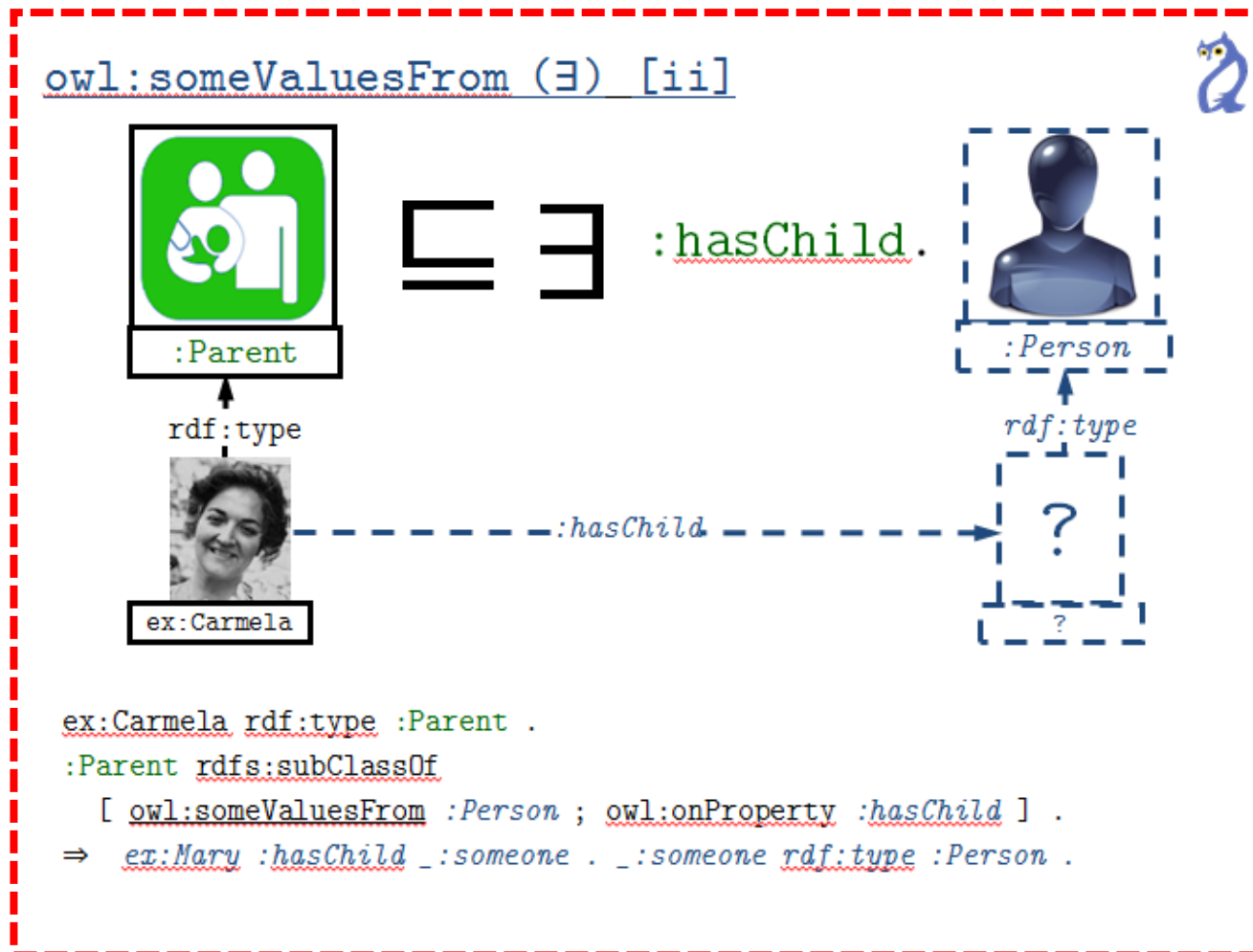
## Incomplete for some Features

|          |                                                                                                                                 |                              |
|----------|---------------------------------------------------------------------------------------------------------------------------------|------------------------------|
| cls-svf1 | $T(?x, \text{owl:someValuesFrom}, ?y)$<br>$T(?x, \text{owl:onProperty}, ?p)$<br>$T(?u, ?p, ?v)$<br>$T(?v, \text{rdf:type}, ?y)$ | $T(?u, \text{rdf:type}, ?x)$ |
|----------|---------------------------------------------------------------------------------------------------------------------------------|------------------------------|



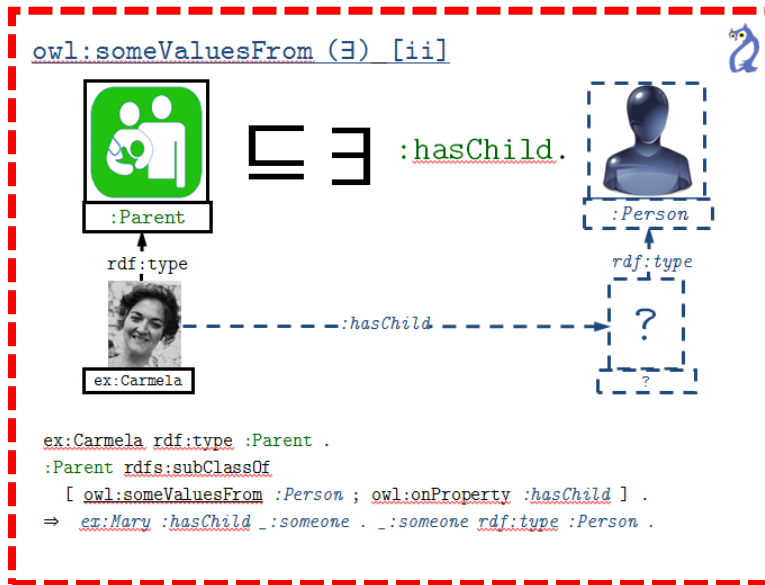
# Why Incomplete?

## Incomplete for some Features



# Why Incomplete?

## Incomplete for some Features



ex:Carmela rdf:type :Parent .

:Parent rdfs:subClassOf

[ owl:someValuesFrom :Person ; owl:onProperty :hasChild ] .

:hasChild rdfs:domain :PostPuberty .

⇒ ex:Carmela rdf:type :PostPuberty .

Will not get this (valid) entailment with OWL 2 RL/RDF

# Why Incomplete?

## Incomplete for some Features

- No support for min-cardinality

```
ex:Carmela :hasChild ex:Sonny , ex:Connie , ex:Fredo , ex:Michael .
ex:Sonny :dateOfBirth "1916-07-23"^^xsd:date .
ex:Connie :dateOfBirth "1922-04-18"^^xsd:date .
ex:Fredo :dateOfBirth "1919-01-08"^^xsd:date .
ex:Michael :dateOfBirth "1920-11-15"^^xsd:date .
:dateOfBirth rdf:type owl:FunctionalProperty .
[ owl:minCardinality 3 ; owl:onProperty :hasChild ] rdfs:subClassOf
  :StressedParent .
⇒ ex:Carmela rdf:type :StressedParent .
```

Will not get this (valid) entailment with OWL 2 RL/RDF

# Why Incomplete?

## Incomplete for some Features

- Limited support for max-cardinality

|           |                                                                                                                                                                 |                                      |
|-----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------|
| cls-maxc1 | <code>T(?x, owl:maxCardinality, "0"^^xsd:nonNegativeInteger)<br/>T(?x, owl:onProperty, ?p)<br/>T(?u, rdf:type, ?x)<br/>T(?u, ?p, ?y)</code>                     | false                                |
| cls-maxc2 | <code>T(?x, owl:maxCardinality, "1"^^xsd:nonNegativeInteger)<br/>T(?x, owl:onProperty, ?p)<br/>T(?u, rdf:type, ?x)<br/>T(?u, ?p, ?y1)<br/>T(?u, ?p, ?y2)</code> | <code>T(?y1, owl:sameAs, ?y2)</code> |

```
ex:Vincent rdf:type :Person ; :hasParent ex:Lucy, ex:Sonny, ex:Santino .
```

```
:Person rdfs:subClassOf [ owl:maxCardinality 2 ; owl:onProperty :hasParent ] .
```

```
ex:Lucy a :Woman . ex:Sonny a :Man . ex:Santino a :Man .
```

```
:Man owl:disjointWith :Woman .
```

```
⇒ ex:Sonny owl:sameAs ex:Santino .
```

Will not get this (valid) entailment with OWL 2 RL/RDF

# Why Incomplete?

Incomplete for some Features

- No support for exact cardinality
- Support also limited for qualified cardinalities

# Why Incomplete?

## Missing Schema Inferences

- Just *some* missing examples for inverse-of:
  - `?a owl:inverseOf ?b .  $\Rightarrow$  ?b owl:inverseOf ?a .`
  - `?a owl:inverseOf ?a .  $\Rightarrow$  ?a rdf:type owl:SymmetricProperty .`
  - `?a rdf:type owl:SymmetricProperty .  $\Rightarrow$  ?a owl:inverseOf ?a .`
  - `?a owl:inverseOf ?b . ?b owl:inverseOf ?a .  
 $\Rightarrow$  ?a owl:equivalentProperty ?b .`
  - `?a owl:inverseOf ?b . ?b rdf:type owl:TransitiveProperty .  
 $\Rightarrow$  ?a rdf:type owl:TransitiveProperty .`
  - `?a owl:inverseOf ?b . ?b rdf:type owl:FunctionalProperty .  
 $\Rightarrow$  ?b rdf:type owl:InverseFunctionalProperty .`
  - `?a owl:inverseOf ?b . ?b rdfs:domain ?c .  
 $\Rightarrow$  ?a rdfs:range ?c .`
  - ...

# Why is OWL 2 RL/RDF Incomplete?

- **Missing features:**
  - `owl:ReflexiveProperty`, `owl:hasSelf`, `owl:minCardinality` ...
- **Problems with disjunction (OR cases)**
  - `owl:unionOf`, `owl:oneOf`, `owl:maxCardinality`, ...
- **Problems with existentials**
  - `owl:someValuesFrom`, `owl:minCardinality`, ...
- **Problems with counting**
  - `owl:minCardinality`, ...
- **Problems with negation**
  - `owl:disjointWith`, `owl:propertyDisjointWith`, `owl:complementOf` ...
- **Incomplete “schema” inferences**

# Finite rules not enough

- Could write a rule for any non-existential case

```
ex:Vincent rdf:type :Person ; :hasParent ex:Lucy, ex:Sonny, ex:Santino .  
:Person rdfs:subClassOf [ owl:maxCardinality 2 ; owl:onProperty :hasParent  
  ] .  
ex:Lucy a :Woman . ex:Sonny a :Man . ex:Santino a :Man .  
:Man owl:disjointWith :Woman .  
⇒ ex:Sonny owl:sameAs ex:Santino .
```

```
?w rdf:type ?c ; ?p ?x , ?y , ?z .  
?c owl:maxCardinality 2 ; owl:onProperty ?p .  
?x owl:differentFrom ?y , ?z .  
⇒ ?y owl:sameAs ?z .
```

- Infinite such rules (have to stop somewhere)

# Existential rules are dangerous

- Could write rules for existential cases too

```
ex:Mary rdf:type :Person .  
:Person rdfs:subClassOf [ owl:someValuesFrom :Person ; owl:onProperty  
    :hasParent ] .  
⇒ ex:Mary :hasParent _:x1 . _:x1 rdf:type :Person .  
_:x1 :hasParent _:x2 . _:x2 rdf:type :Person .  
... ∞
```

```
?x rdf:type ?c .  
?c owl:someValuesFrom ?d ; owl:onProperty ?p .  
⇒ ?x ?p _:b . _:b rdf:type ?d .
```

- Might lead to materialising  $\infty$  entailments
  - (In this case if `?x rdf:type ?d .`  $\Rightarrow$  `?x rdf:type ?c .`)

**COMPLETE REASONERS  
THAT MAY NOT HALT**

# Complete reasoners that may not halt:

Quite Practical!

- **Cons:**

- Erm ... reasoner may never halt

What might the “pros” be in this case?

- **Pros:**

- Avoid complicated decidability restrictions!

Imagine restricting C or Java to be decidable

1. Don't allow features like loops/recursion
  - But not all programs with loops/recursion fail to halt!
2. Restrict how features like loops/recursion can be used
  - More detailed restrictions allow more programmes but are more complicated to understand ☹

# Complete reasoners that may not halt:

Rare in practice

- Only line of work on this I know of:

## Reasoning in the OWL 2 Full Ontology Language using First-Order Automated Theorem Proving

Michael Schneider<sup>1\*</sup> and Geoff Sutcliffe<sup>2</sup>

<sup>1</sup> FZI Research Center for Information Technology, Germany

<sup>2</sup> University of Miami, USA

**Abstract.** OWL 2 has been standardized by the World Wide Web Consortium (W3C) as a family of ontology languages for the Semantic Web. The most expressive of these languages is OWL 2 Full, but to date no reasoner has been implemented for this language. Consistency and entailment checking are known to be undecidable for OWL 2 Full. We have translated a large fragment of the OWL 2 Full semantics into first-order logic, and used automated theorem proving systems to do reasoning based on this theory. The results are promising, and indicate that this approach can be applied in practice for effective OWL reasoning, beyond the capabilities of current Semantic Web reasoners.

This is an *extended version* of a paper with the same title that has been published at CADE 2011, LNAI 6803, pp. 446–460. The extended version provides appendices with additional resources that were used in the reported evaluation.

**Key words:** Semantic Web, OWL, First-order logic, ATP

*not going to talk  
about this but  
good to know  
about! 😊*

## 1 Introduction

The Web Ontology Language OWL 2 [16] has been standardized by the World

**RESTRICT OWL TO  
GUARANTEE DECIDABILITY**

# Recap ...

- **Accept incomplete reasoners that halt**
  - Complete language, incomplete reasoning, halts
- **Accept complete reasoners that may not halt**
  - Complete language, complete reasoning, may not halt
- **Restrict OWL so reasoning becomes decidable**
  - Restricted language, complete reasoning, halts

## Core idea:

Restrict OWL so that complete reasoning is decidable over any ontology written within those restrictions

# Restrict OWL to guarantee decidability:

## How to guarantee decidability?

- We've seen how to prove that something is undecidable

How can we prove that something is decidable?

- Most commonly: give an algorithm that halts ...

Problem: Consecutive '1's in  $\pi$

- **Input:** A natural number  $n$
- **Output:**
  - true if  $\pi$  contains  $n$  consecutive '1's
  - false otherwise

Is this problem **DECIDABLE** or **UNDECIDABLE**?

... i.e., does there exist a program that halts (with the correct answer) for all  $n$ ?

What if we knew the maximum sequence of consecutive '1's in  $\pi$ ?

```
if (n ≤ MAX) return true; else return false;
```

- there must exist a MAX sequence of consecutive '1's in  $\pi$  (even if it's  $\infty$ )
- ∴ there must exist a correct program that halts (even if we don't know its details)
- ∴ problem is **DECIDABLE**!

# Restrict OWL to guarantee decidability:

How to guarantee decidability?

- Focus on satisfiability/entailment checking
  - Recall: Can (usually) reduce entailment to satisfiability
    - (So long as we can do negation in the language)

How can we perform reasoning?

- Does Ontology  $O$  entail  $O'$ ?

```
:hasDaughter rdfs:subPropertyOf :hasChild .  
ex:Michael :hasDaughter ex:Mary .
```

```
ex:Michael :hasChild ex:Mary .
```

- Could instead ask: is “ $O \cup \neg O'$ ” unsatisfiable?

```
:hasDaughter rdfs:subPropertyOf :hasChild .  
ex:Michael :hasDaughter ex:Mary .  
[] owl:sourceIndividual ex:Michael ;  
   owl:assertionProperty :hasChild ;  
   owl:targetIndividual ex:Mary .
```

Can reduce entailment to unsatisfiability!

# Restrict OWL to guarantee decidability:

## Sublanguages of OWL 2

- **Description Logic community**
  - Predates OWL
  - Looks at decidable subsets of First Order Logic
  - Results can be applied to OWL!
- **OWL 2 Full**: The unrestricted, undecidable language
- **OWL 2 DL**: A restricted, decidable version

# Restrict OWL to guarantee decidability:

## Sublanguages of OWL 2

- What is restricted?

What's the entailment question?

$$D \equiv D_1 \sqcup D_2 \sqcup \dots \sqcup D_{k-1} \sqcup D_k$$

$$D_i \sqcap D_j \sqsubseteq \perp \text{ (for } 1 \leq i < j \leq k \text{)}$$

$$D \sqsubseteq (\exists r.D) \sqcap (\exists a.D)$$

$$D_1 \sqsubseteq \forall r. \left( \bigsqcup_{D' \in R(D_1)} D' \right) \sqcap \forall a. \left( \bigsqcup_{D' \in A(D_1)} D' \right)$$

...

$$D_k \sqsubseteq \forall r. \left( \bigsqcup_{D' \in R(D_k)} D' \right) \sqcap \forall a. \left( \bigsqcup_{D' \in A(D_k)} D' \right)$$

$$d \sqsubseteq a \circ r, \quad d \sqsubseteq r \circ a, \quad \text{Func}(d)$$

← Must restrict something here  
(for example)

$$D \equiv \perp$$

**Goal:** Ontology  $\mathcal{O}$  entails  $\mathcal{O}'$  if and only if  $D$  has an infinite tiling

If  $D$  has any member (a “tile”), it must have an infinite tiling!

If  $D$  has no member, it must not have an infinite tiling.

# Restrict OWL to guarantee decidability:

## Sublanguages of OWL 2

- What is restricted?

What's the entailment question?

$$D \equiv D_1 \sqcup D_2 \sqcup \dots \sqcup D_{k-1} \sqcup D_k$$
$$D_i \sqcap D_j \sqsubseteq \perp \text{ (for } 1 \leq i < j \leq k)$$
$$D \sqsubseteq (\exists r.D) \sqcap (\exists a.D)$$
$$D_1 \sqsubseteq \forall r. \left( \bigsqcup_{D' \in R(D_1)} D' \right) \sqcap \forall a. \left( \bigsqcup_{D' \in A(D_1)} D' \right) \quad \leftarrow \text{Must restrict something here}$$
$$\dots$$
$$D_k \sqsubseteq \forall r. \left( \bigsqcup_{D' \in R(D_k)} D' \right) \sqcap \forall a. \left( \bigsqcup_{D' \in A(D_k)} D' \right)$$
$$d \sqsubseteq a \circ r, \quad d \sqsubseteq r \circ a, \quad \text{Func}(d)$$
$$D \equiv \perp$$

Goal: Ontology  $\mathcal{O}$  entails  $\mathcal{O}'$  if and only if  $D$  has an infinite tiling  
If  $D$  has any member (a "tile"), it must have an infinite tiling!  
If  $D$  has no member, it must not have an infinite tiling.

- For example, OWL 2 DL restricts:
  - functional properties to be "simple" (no chains, no transitivity)

# Restrict OWL to guarantee decidability:

## Sublanguages of OWL 2

- What is restricted?
- For example, OWL 2 DL restricts:
  - functional properties to be “simple” (no chains, no transitivity)
  - likewise properties used with hasSelf, cardinalities, inverse functionality, asymmetry and irreflexivity must be simple
  - inverse functional properties must be object properties
  - need to follow specific RDF syntax and explicitly declare classes, object properties (with IRI values), datatype properties (with literal values)
  - ... more (it's really quite messy ☹)

# Restrict OWL to guarantee decidability:

On the plus side ...

- OWL 2 DL still supports **disjunction**,  
existentials, counting, **negation**!

$O \models O' ?$

```
ex:Vincent rdf:type :Person , :Godfather .  
:Godfather owl:disjointWith :Woman .  
:Person owl:equivalentClass [ owl:unionOf ( :Woman :Man ) ] .
```

```
 $\models ex:Vincent \text{ rdf:type } :Man .$ 
```

# Restrict OWL to guarantee decidability:

On the plus side ...

- OWL 2 DL still supports disjunction, **existentials**, counting, negation!

$O \models O' ?$

```
ex:Carmela rdf:type :Parent .
```

```
:Parent rdfs:subClassOf
```

```
[ owl:someValuesFrom :Person ; owl:onProperty :hasChild ] .
```

```
:hasChild rdfs:domain :PostPuberty .
```

```
⊨ ex:Carmela rdf:type :PostPuberty .
```



# Restrict OWL to guarantee decidability:

On the plus side ...

- OWL 2 DL still supports disjunction, existentials, **counting**, negation!

$O \models O' ?$

```
ex:Carmela :hasChild ex:Sonny , ex:Connie , ex:Fredo , ex:Michael .
ex:Sonny :dateOfBirth "1916-07-23"^^xsd:date .
ex:Connie :dateOfBirth "1922-04-18"^^xsd:date .
ex:Fredo :dateOfBirth "1919-01-08"^^xsd:date .
ex:Michael :dateOfBirth "1920-11-15"^^xsd:date .
:dateOfBirth rdf:type owl:FunctionalProperty .
[ owl:minCardinality 3 ; owl:onProperty :hasChild ] rdfs:subClassOf
  :StressedParent .
```

$\models ex:Carmela \text{ rdf:type } :StressedParent$  .

So long as  $O$  and  $O'$  follow the OWL 2 DL restrictions,  
you are guaranteed a correct answer to  $O \models O'!$

# Restrict OWL to guarantee decidability:

An algorithm for OWL 2 DL

- What sort of algorithm can we use?
- One answer: Tableau (positive *sketch* below)

```
ex:Vincent rdf:type :Person , :Godfather .  
:Godfather owl:disjointWith :Woman .  
:Person owl:equivalentClass [ owl:unionOf ( :Woman :Man )  
⊨ ex:Vincent rdf:type :Man .
```

$O \models O' ?$

# Restrict OWL to guarantee decidability:

An algorithm for OWL 2 DL

- What sort of algorithm can we use?
- One answer: Tableau (positive *sketch* below)

```
ex:Vincent rdf:type :Person , :Godfather .  
:Godfather owl:disjointWith :Woman .  
:Person owl:equivalentClass [ owl:unionOf ( :Woman :Man )  
    ] .  
- ex:Vincent rdf:type :Man .
```

Unsatisfiable?



Branch for OR



```
ex:Vincent rdf:type :Person , :Godfather  
.  
:Godfather owl:disjointWith :Woman .  
ex:Vincent rdf:type Man .  
- ex:Vincent rdf:type :Man
```



```
ex:Vincent rdf:type :Person , :Godfather  
.  
:Godfather owl:disjointWith :Woman .  
ex:Vincent rdf:type Woman .  
- ex:Vincent rdf:type :Man
```



Unsatisfiable in all branches  $\rightarrow O \cup \neg O'$  unsatisfiable  $\rightarrow O \models O'$

# Restrict OWL to guarantee decidability:

An algorithm for OWL 2 DL

- What sort of algorithm can we use?
- One answer: Tableau (negative *sketch* below)

```
ex:Vincent rdf:type :Person , :Godfather .  
:Godfather owl:disjointWith :Woman .  
:Person owl:equivalentClass [ owl:unionOf ( :Woman :Man )  
⊨ ex:Vincent rdf:type :Woman .
```

$O \models O' ?$

# Restrict OWL to guarantee decidability:

An algorithm for OWL 2 DL

- What sort of algorithm can we use?
- One answer: Tableau (negative *sketch* below)

```
ex:Vincent rdf:type :Person , :Godfather .  
:Godfather owl:disjointWith :Woman .  
:Person owl:equivalentClass [ owl:unionOf ( :Woman :Man )  
    ] .  
-ex:Vincent rdf:type :Woman .
```

Unsatisfiable?



Branch for OR



```
ex:Vincent rdf:type :Person , :Godfather .  
:Godfather owl:disjointWith :Woman .  
ex:Vincent rdf:type Man .  
-ex:Vincent rdf:type :Woman .
```

OKAY

```
ex:Vincent rdf:type :Person , :Godfather .  
:Godfather owl:disjointWith :Woman .  
ex:Vincent rdf:type Woman .  
-ex:Vincent rdf:type :Woman .
```



Satisfiable in a branch  $\rightarrow O \cup \neg O'$  satisfiable  $\rightarrow O \not\models O'$

# Restrict OWL to guarantee decidability:

An algorithm for OWL 2 DL!

- We have a complete algorithm that halts and that supports a lot of the OWL features!



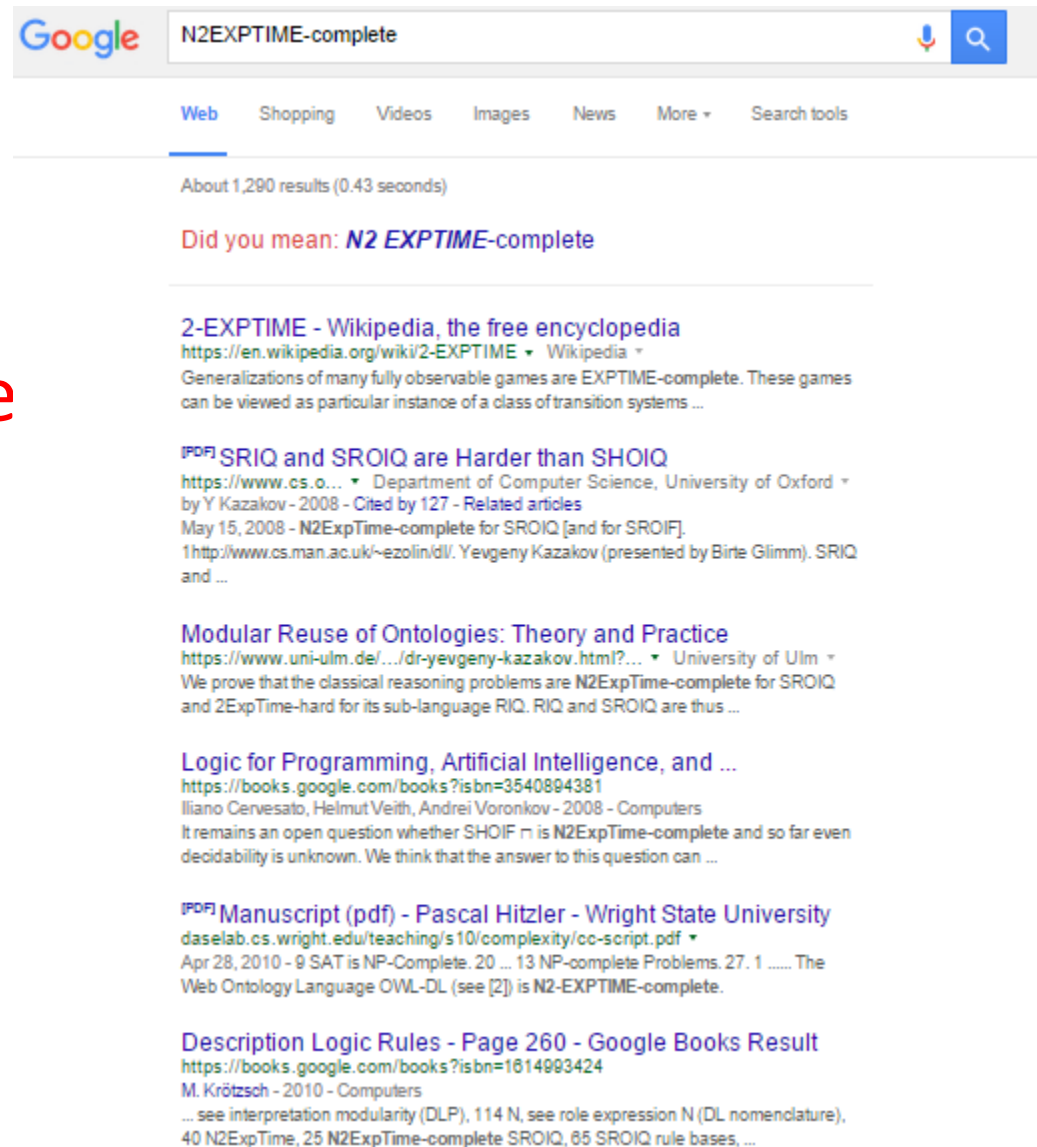
# Restrict OWL to guarantee decidability:

An algorithm for OWL 2 DL!

- A few problems:
  - We have to give the entailments to check
    - Cannot just ask to compute the entailments
  - Restrictions are complicated
    - Very complicated
    - And often are broken by real-world ontologies
  - Tableau reasoning is really expensive
    - Branch for every disjunction suggests exponential
    - N2EXPTIME-complete (!!?!!!)
      - $2^{2^n}$  on a non-deterministic machine
      - ...

# N2EXPTIME-Complete so nasty ...

The only results  
returned by Google  
relate to OWL



Google N2EXPTIME-complete

Web Shopping Videos Images News More Search tools

About 1,290 results (0.43 seconds)

Did you mean: **N2 EXPTIME**-complete

**2-EXPTIME - Wikipedia, the free encyclopedia**  
<https://en.wikipedia.org/wiki/2-EXPTIME> • Wikipedia  
Generalizations of many fully observable games are EXPTIME-complete. These games can be viewed as particular instance of a class of transition systems ...

**[PDF] SRIQ and SROIQ are Harder than SHOIQ**  
[https://www.cs.ox.ac.uk/~dpc/Department of Computer Science, University of Oxford](https://www.cs.ox.ac.uk/~dpc/Department%20of%20Computer%20Science,%20University%20of%20Oxford%20-%20Y%20Kazakov%20-%202008) • by Y Kazakov - 2008 - Cited by 127 - Related articles  
May 15, 2008 - N2ExpTime-complete for SROIQ [and for SROIF].  
1 <http://www.cs.man.ac.uk/~ezolin/dl/>. Yevgeny Kazakov (presented by Birte Glimm). SRIQ and ...

**Modular Reuse of Ontologies: Theory and Practice**  
<https://www.uni-ulm.de/~dpc/dr-yevgeny-kazakov.html?...> • University of Ulm  
We prove that the classical reasoning problems are N2ExpTime-complete for SROIQ and 2ExpTime-hard for its sub-language RIQ. RIQ and SROIQ are thus ...

**Logic for Programming, Artificial Intelligence, and ...**  
<https://books.google.com/books?isbn=3540894381>  
Ilario Cervesato, Helmut Veith, Andrei Voronkov - 2008 - Computers  
It remains an open question whether SHOIF is N2ExpTime-complete and so far even decidability is unknown. We think that the answer to this question can ...

**[PDF] Manuscript (pdf) - Pascal Hitzler - Wright State University**  
[daseelab.cs.wright.edu/teaching/s10/complexity/cc-script.pdf](https://daseelab.cs.wright.edu/teaching/s10/complexity/cc-script.pdf) •  
Apr 28, 2010 - 9 SAT is NP-Complete. 20 ... 13 NP-complete Problems. 27. 1 ..... The Web Ontology Language OWL-DL (see [2]) is N2-EXPTIME-complete.

**Description Logic Rules - Page 260 - Google Books Result**  
<https://books.google.com/books?isbn=1614993424>  
M. Krötzsch - 2010 - Computers  
... see interpretation modularity (DLP), 114 N, see role expression N (DL nomenclature), 40 N2ExpTime, 25 N2ExpTime-complete SROIQ, 65 SROIQ rule bases, ...

# N2EXPTIME-Complete (OWL 2 DL's small print) ...

- Checking entailment is guaranteed to halt for OWL 2 DL restricted ontologies\*

\* halt may not occur before heat death of the universe



# OWL 2 DL performance considerations

- Not all OWL 2 DL ontologies will run into worst-cases
- Entailments will work fine for most small ontologies
- Scalability still a real issue in practice

# OWL 2 Profiles (briefly)

- **More efficient sublanguages of OWL 2 DL**
  - More restrictions to allow complete reasoning with more efficient algorithms
- **OWL 2 RL**: A restriction of OWL 2 DL such that OWL 2 RL/RDF rules provide complete reasoning (in some sense we won't get into)
- **OWL 2 EL**: Tractable algorithm for classifying ontologies
- **OWL 2 QL**: Tractable algorithm based on rewriting SQL queries

**IMPRESSIONS ...**

Opinion of lots of people in the Semantic Web  
with respect to OWL ...



Also perhaps part of the reason why you see things like ...



# Is OWL good for the Semantic Web?

- It provides formal foundations for semantics
- Indicates what's possible, what's not with respect to machine-readable semantics
  - What's efficient, what's not
- Offers options: OWL 2 RL/EL/QL/DL/Full
- Drives many applied/practical people crazy
- Some theoretical folks also consider it to have poor aesthetic
- Makes lots of bad assumptions for the Web
  - Not scalable
  - Strict in what it accepts
  - Blindly accepting

What do you think?

If we have time ...

Let's model a domain ...

**RECAP ...**

# Coping with undecidability (reasoning) ...

- **Accept incomplete reasoners that halt**
  - Complete language, incomplete reasoning, halts
  - e.g., OWL 2 RL/RDF rules can be applied on any RDF data using any OWL features in any way, but may not get all inferences
- **Accept complete reasoners that may not halt**
  - Complete language, complete reasoning, may not halt
  - e.g., can use a first-order-theorem prover, but it may run forever on some input ontologies
- **Restrict OWL so reasoning becomes decidable**
  - Restricted language, complete reasoning, halts
  - e.g., can restrict the OWL 2 Full language to sublanguages that have decidable/tractable reasoning algorithms

# OWL 2 RL/RDF rules

- What we've been using in the labs
- Rules supporting a lot of OWL
  - but incomplete
- Can be run over any RDF/OWL data
  - no restrictions needed!
- Can materialise entailments
- Relatively efficient in practice
- Easy to implement, not so hard to understand

# OWL 2 Full / Complete reasoning

- Not a lot of work
- One proposal using a First-Order-Logic theorem prover

# OWL 2 DL

- Restrict OWL 2 Full to make entailment/satisfiability checking decidable
- Complete reasoning with respect to ontologies following restrictions
  - Supports some pretty complex entailments
  - Will always halt with a correct answer eventually
- Very bad worst-case: 2NEXPTIME-Complete
  - May not halt before end of universe
  - Worst-cases might be rare, but scalability and compute times still often encountered in practice
- Need to ask if something specific is entailed
  - Cannot materialise “all” entailments
- Restrictions make the whole thing nasty to understand

# OWL 2 Profiles

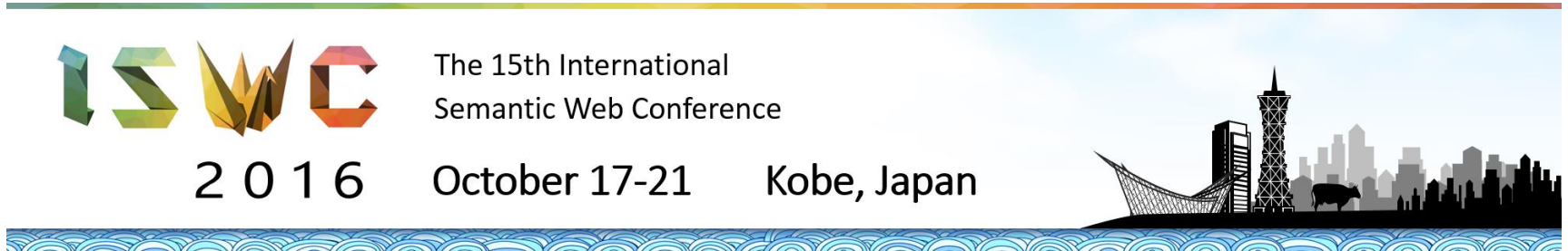
- **More efficient sublanguages of OWL 2 DL**
  - More restrictions to allow complete reasoning with more efficient algorithms
- **OWL 2 RL**: A restriction of OWL 2 DL such that OWL 2 RL/RDF rules provide complete reasoning (in some sense)
- **OWL 2 EL**: Tractable algorithm for classifying ontologies
- **OWL 2 QL**: Tractable algorithm based on rewriting SQL queries

# End of main OWL part (after next lab)



... rest of material should be easier / more applied  
(but I hope you learned something about why telling  
machines stuff about the world is hard)

# No lecture/lab *next* week (Oct. 17/19)



Will post an assignment in the forum. 😊

Questions?

