CC6202-1
La Web de Datos
Primavera 2015

Lecture 5: Web Ontology Language (II)

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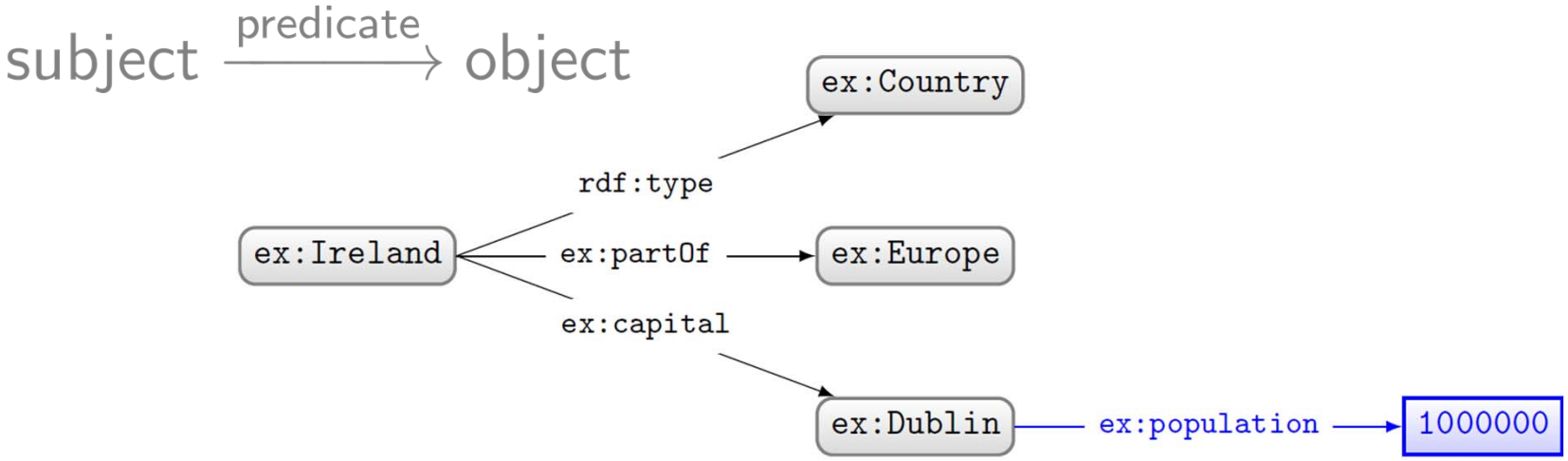
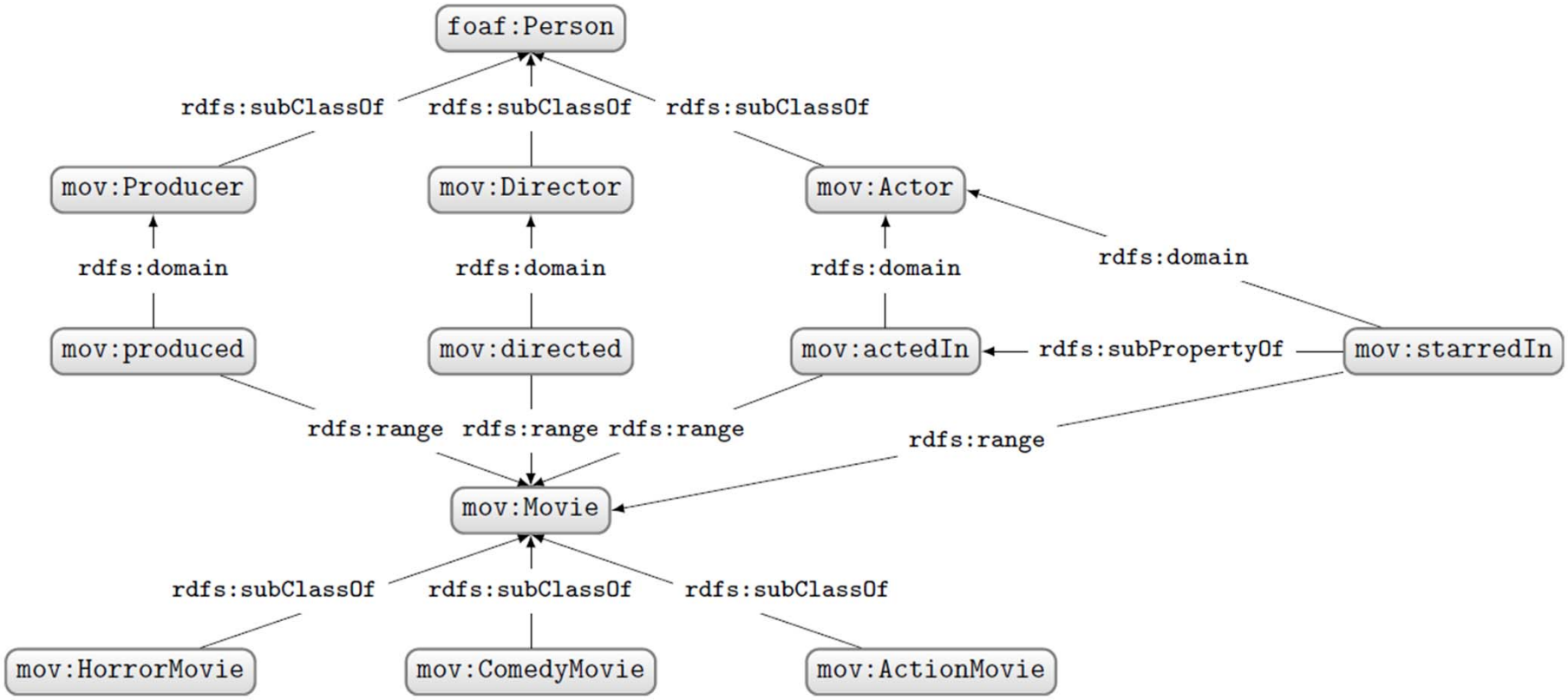
PREVIOUSLY ON
“LA WEB DE DATOS”

RDF: Resource Description Framework

subject	*predicate*	*object*
ex:Ireland	ex:partOf	ex:Europe
ex:Ireland	rdf:type	ex:Country
ex:Ireland	ex:capital	ex:Dublin
ex:Dublin	ex:population	1,000,000

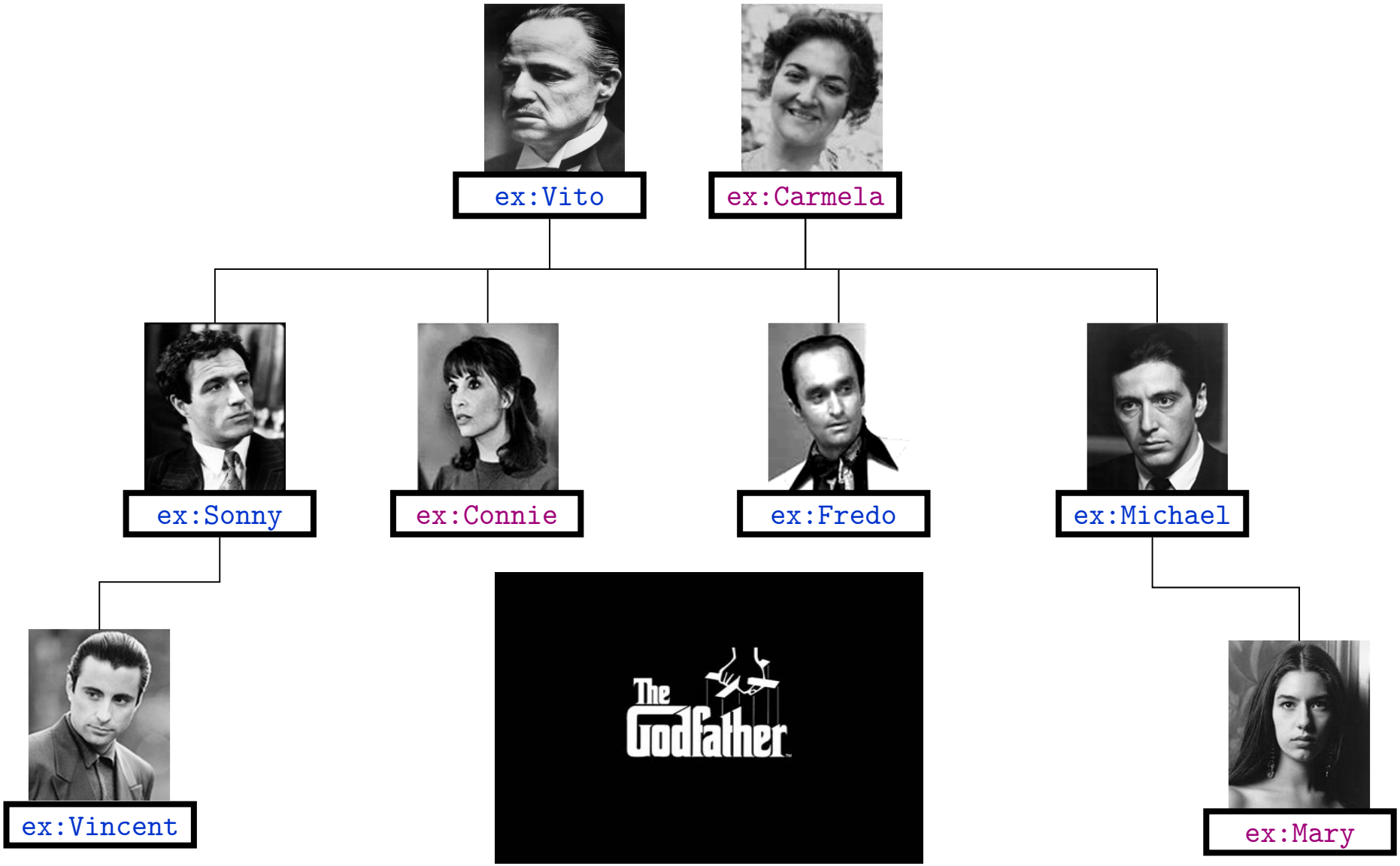


RDF Schema …

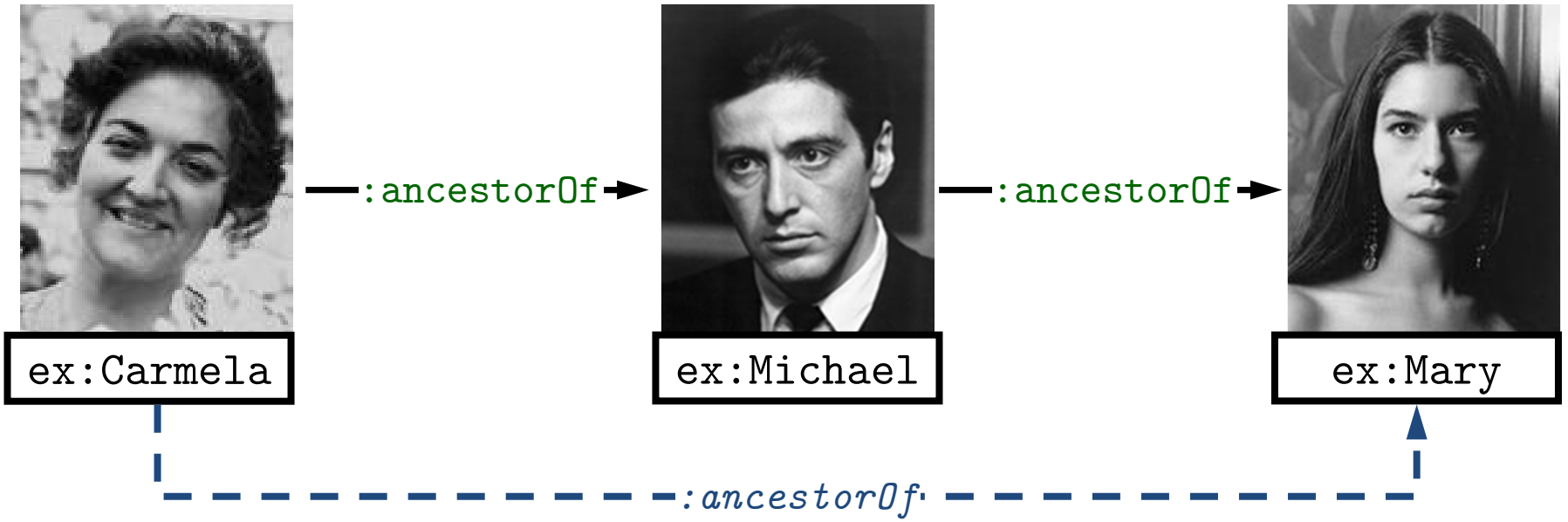


(*an example*)

Modelling family relations with OWL

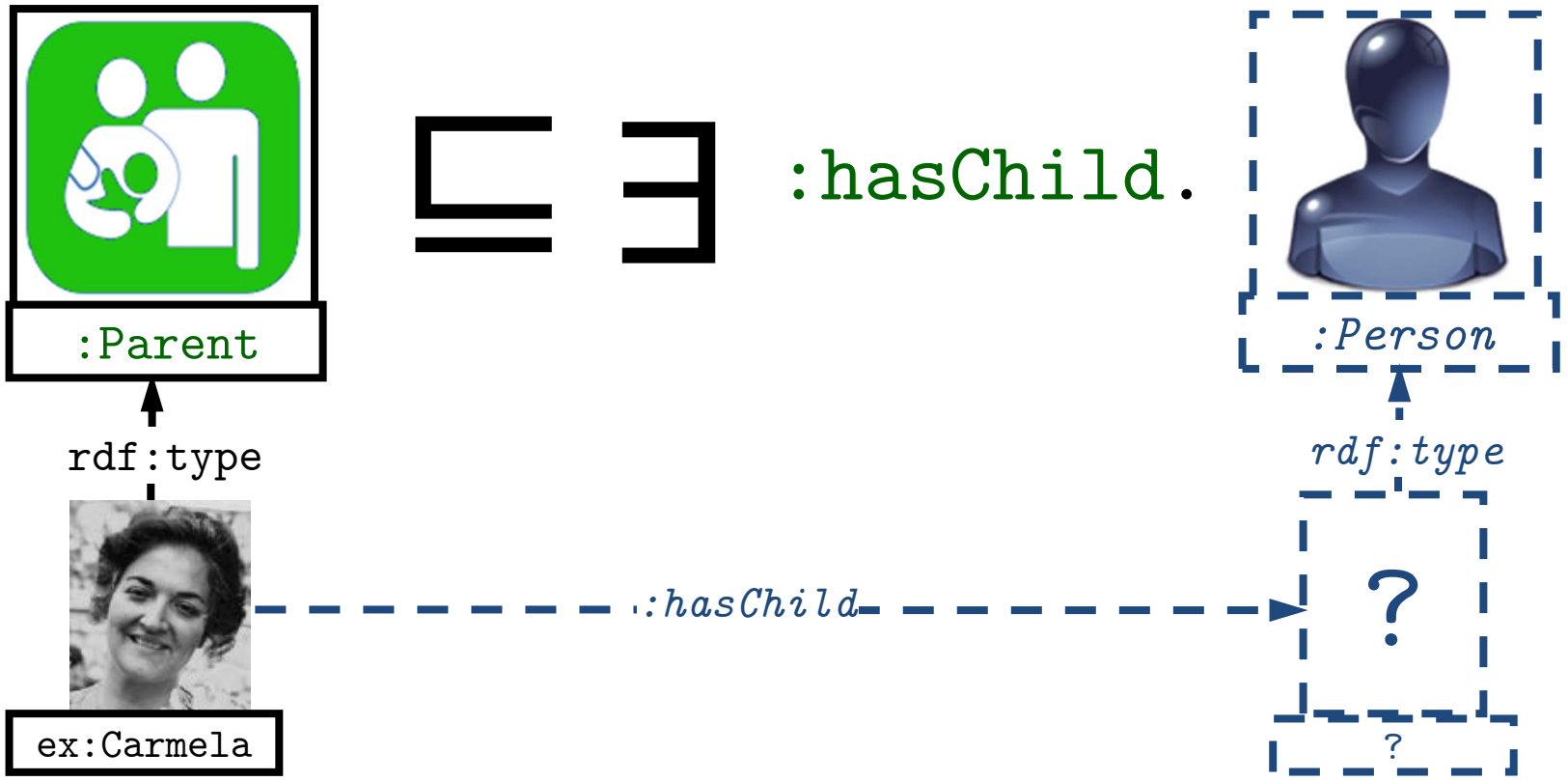


owl:TransitiveProperty

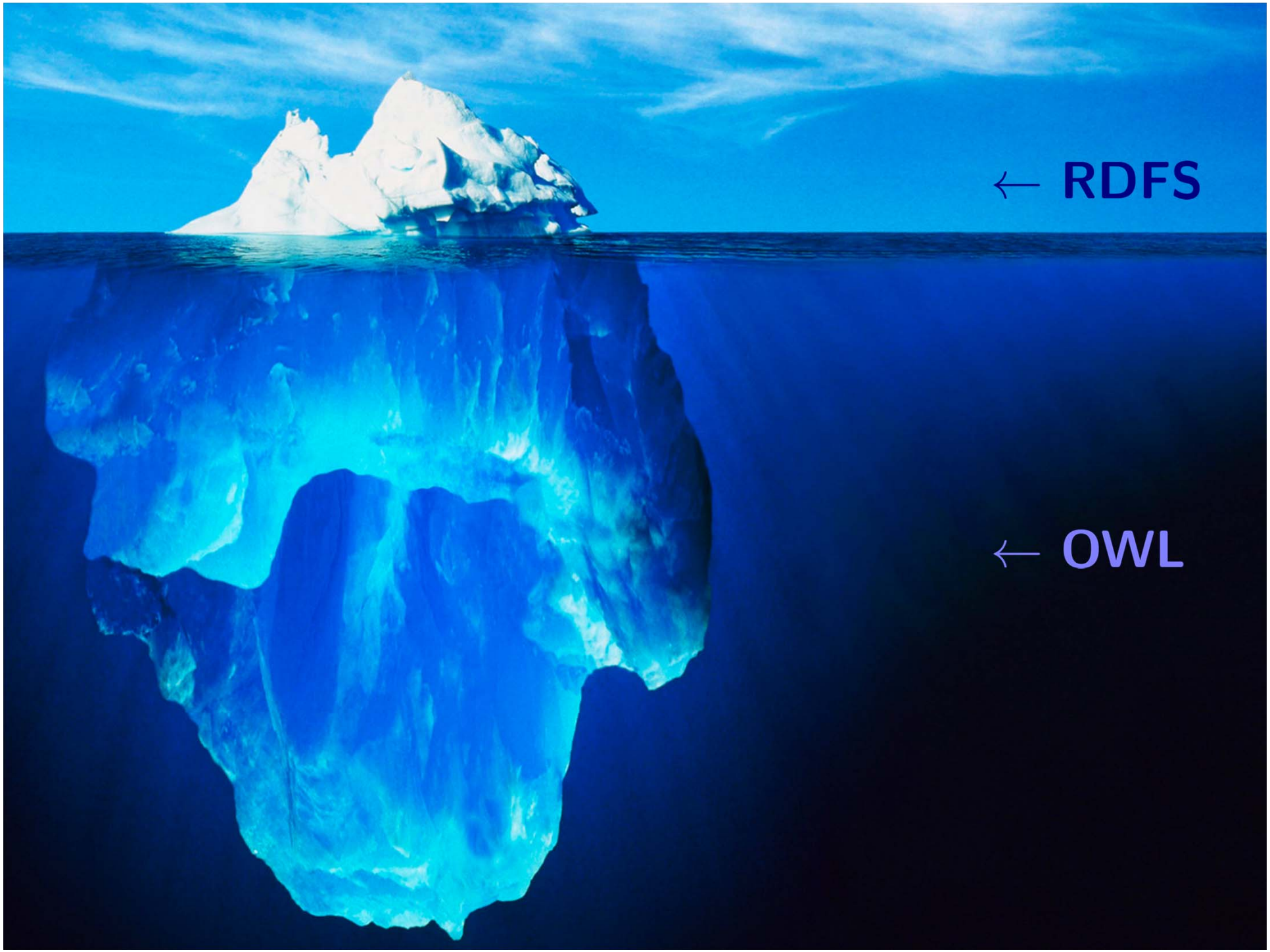



```
ex:Carmela :ancestorOf ex:Michael .
ex:Michael :ancestorOf ex:Mary .
:ancestorOf rdf:type owl:TransitiveProperty .
⇒  ex:Carmela :ancestorOf ex:Mary .
```

owl:someValuesFrom (∃) [ii]




```
ex:Carmela rdf:type :Parent .
:Parent rdfs:subClassOf
  [ owl:someValuesFrom :Person ; owl:onProperty :hasChild ] .
⇒  ex:Mary :hasChild _:someone . _:someone rdf:type :Person .
```


← RDFS
← OWL

TODAY’S TOPIC …

An ontology is just some definitions …

```
:hasDaughter rdfs:subPropertyOf :hasChild .
:hasChild rdf:type owl:AsymmetricProperty ; owl:inverseOf :hasParent ;
  rdfs:subPropertyOf :ancestorOf .
:ancestorOf rdf:type owl:TransitiveProperty .
ex:Carmela :hasChild ex:Michael .
ex:Michael :hasDaughter ex:Mary .
ex:Mary a :Person .
:Person owl:equivalentClass
  [ owl:qualifiedCardinality 2 ;
    owl:onProperty :hasParent ;
    owl:onClass :Person ] .
```

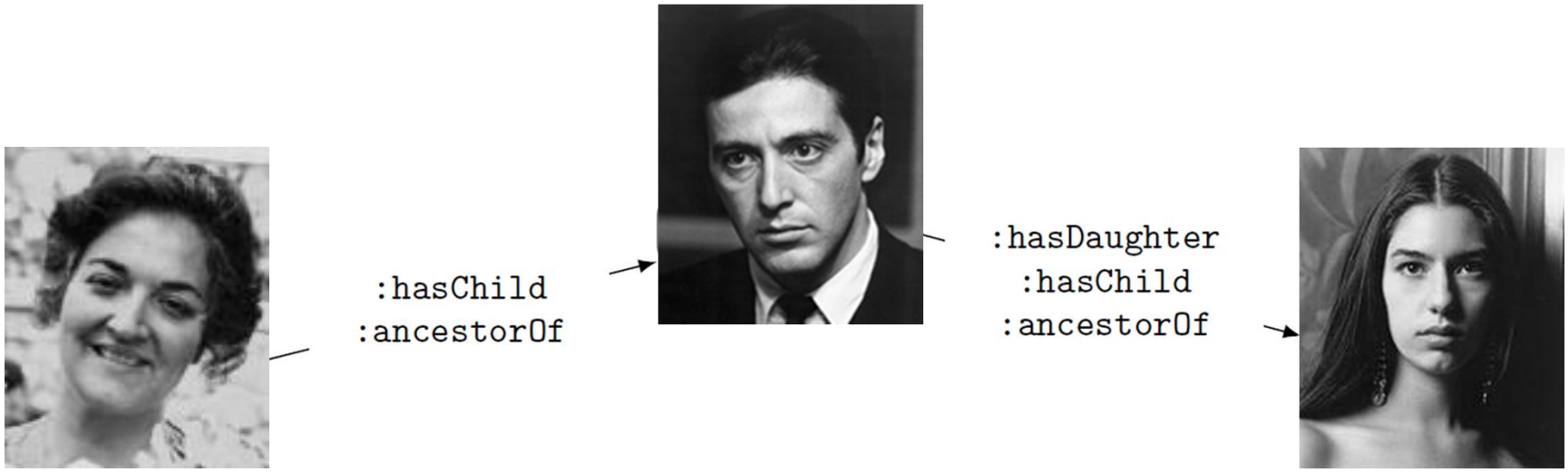
… but what do they mean?
… and what can we do with ontologies?

MODELS …

Models of ontologies

A model is any world that an ontology might describe

```
:hasDaughter rdfs:subPropertyOf :hasChild .
:hasChild rdfs:subPropertyOf :ancestorOf .
:ancestorOf rdf:type owl:TransitiveProperty .
ex:Carmela :hasChild ex:Michael .
ex:Michael :hasDaughter ex:Mary .
```

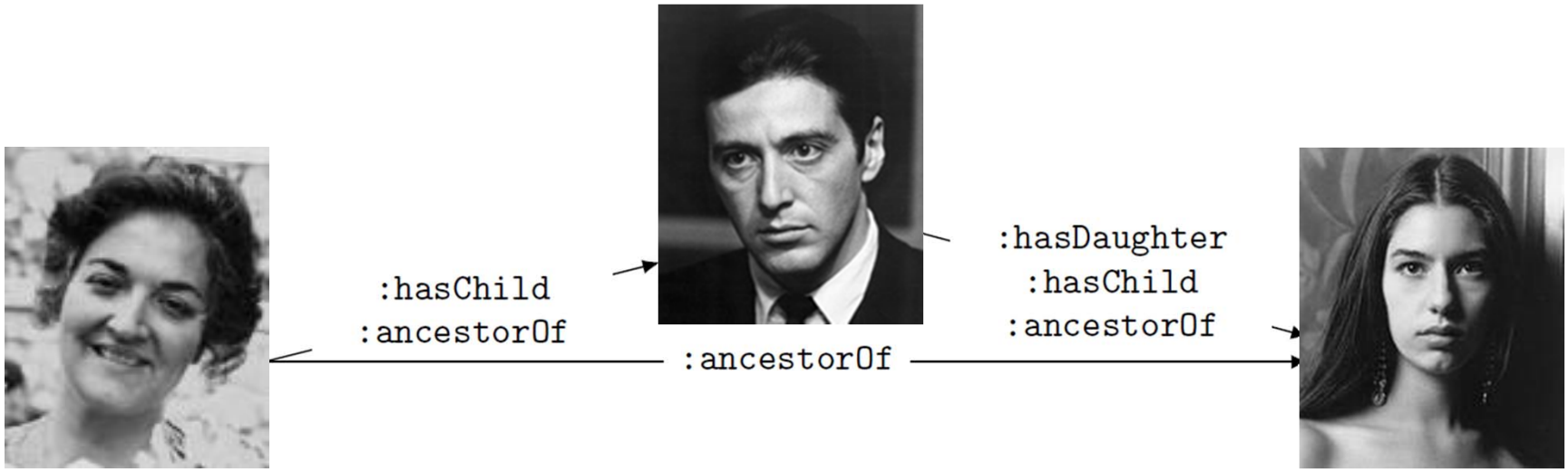


… not a model, since Carmela would need to be an ancestor of Mary

Models of ontologies

A model is any world that an ontology might describe

```
:hasDaughter rdfs:subPropertyOf :hasChild .
:hasChild rdfs:subPropertyOf :ancestorOf .
:ancestorOf rdf:type owl:TransitiveProperty .
ex:Carmela :hasChild ex:Michael .
ex:Michael :hasDaughter ex:Mary .
```



… a model

(leaving aside things like OWL definitions, reflexive `owl:sameAs`, etc.)

Models of ontologies

A model is any world that an ontology might describe

```
:hasDaughter rdfs:subPropertyOf :hasChild .
:hasChild rdfs:subPropertyOf :ancestorOf .
:ancestorOf rdf:type owl:TransitiveProperty .
ex:Carmela :hasChild ex:Michael .
ex:Michael :hasDaughter ex:Mary .
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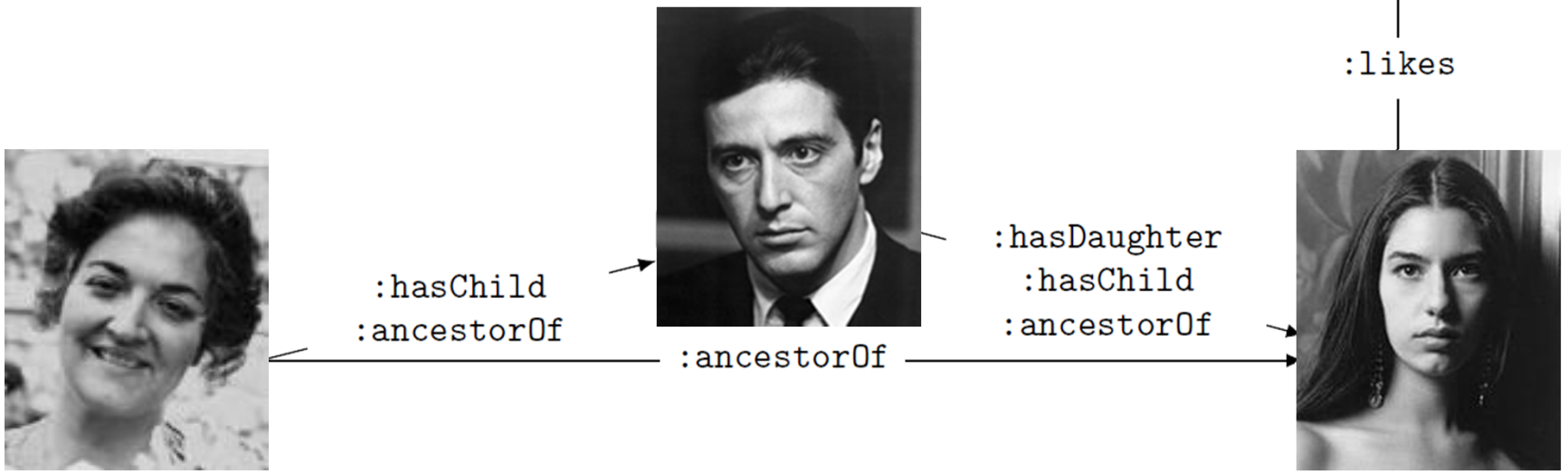


… also a model

(Under Open World Assumption, Ontology can describe part of the world)

Models of ontologies

A model is any world that an ontology might describe

```
:hasDaughter rdfs:subPropertyOf :hasChild .
:hasChild rdfs:subPropertyOf :ancestorOf .
:ancestorOf rdf:type owl:TransitiveProperty .
ex:Carmela :hasChild ex:Michael .
ex:Michael :hasDaughter ex:Mary .
```

:likes

:hasChild
:ancestorOf

:ancestorOf

:hasDaughter
:hasChild
:ancestorOf

… also a model

(Since we don't know what `ex:Cake`, `ex:Carmela` , etc., actually refer to)

Mapping of names to things part of model

ex:Cake

ex:Michael

ex:Carmela

ex:Mary

(Different names to different things means a different model!)

ENTAILMENT …

Ontology O entails O' ($O \models O'$)

```
:hasDaughter rdfs:subPropertyOf :hasChild .
:hasChild rdf:type owl:AsymmetricProperty ; owl:inverseOf :hasParent ;
  rdfs:subPropertyOf :ancestorOf .
:ancestorOf rdf:type owl:TransitiveProperty .
ex:Carmela :hasChild ex:Michael .
ex:Michael :hasDaughter ex:Mary .
ex:Mary a :Person .
:Person owl:equivalentClass
  [ owl:qualifiedCardinality 2 ;
    owl:onProperty :hasParent ;
    owl:onClass :Person ] .
```

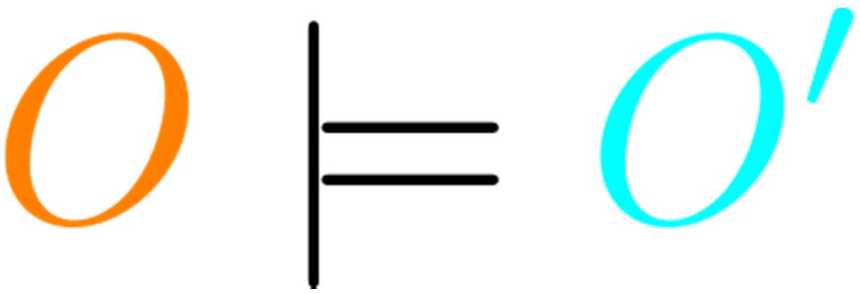
```
ex:Michael :hasParent ex:Carmela .
ex:Michael :hasChild ex:Mary .
ex:Carmela :ancestorOf ex:Mary .
```

Any model of O is a model of O'

(O' forms part of the models of O)

(O' says nothing new over O)

Entailment symbol: ⊨



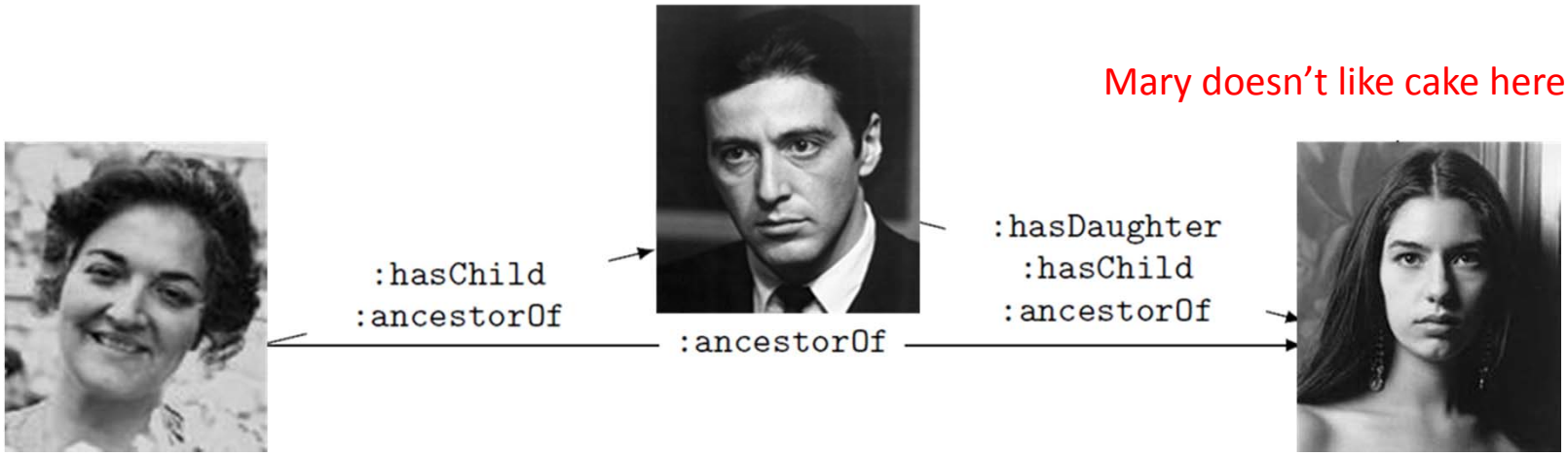
Ontology Entailment

```
ex:Michael :hasParent ex:Carmela .
ex:Michael :hasChild ex:Mary .
ex:Carmela :ancestorOf ex:Mary .
```

```
ex:Carmela :ancestorOf ex:Mary .
ex:Mary :likes ex:Cake .
```

Does O entail O' ($O \models O'$)?

No! There are models of O that are not of O' …



REASONING TASKS …

Materialisation:

Write down entailments

```
:hasDaughter rdfs:subPropertyOf :hasChild .
:hasChild rdf:type owl:AsymmetricProperty ; owl:inverseOf :hasParent ;
  rdfs:subPropertyOf :ancestorOf .
:ancestorOf rdf:type owl:TransitiveProperty .
ex:Carmela :hasChild ex:Michael .
ex:Michael :hasDaughter ex:Mary .
ex:Mary a :Person .
:Person owl:equivalentClass
  [ owl:qualifiedCardinality 2 ;
    owl:onProperty :hasParent ;
    owl:onClass :Person ] .
```

```
ex:Michael :hasParent ex:Carmela .
ex:Michael :hasChild ex:Mary .
ex:Carmela :ancestorOf ex:Mary .
…
```

Any problems with this?

Materialisation:

Write down entailments

```
:hasDaughter rdfs:subPropertyOf :hasChild .
:hasChild rdf:type owl:AsymmetricProperty ; owl:inverseOf :hasParent ;
  rdfs:subPropertyOf :ancestorOf .
:ancestorOf rdf:type owl:TransitiveProperty .
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ex:Mary a :Person .
:Person owl:equivalentClass
  [ owl:qualifiedCardinality 2 ;
    owl:onProperty :hasParent ;
    owl:onClass :Person ] .
```

```
ex:Michael :hasParent ex:Carmela .
ex:Michael :hasChild ex:Mary .
ex:Carmela :ancestorOf ex:Mary .
```

```
ex:Mary :hasParent _:parent1 . _:parent1 a :Person .
ex:Mary :hasParent _:parent2 . _:parent2 a :Person .
_:parent1 :hasParent _:parent11 . _:parent11 a :Person .
_:parent2 :hasParent _:parent12 . _:parent12 a :Person . ...
```

Materialisation:

Write down entailments

```
:hasDaughter rdfs:subPropertyOf :hasChild .
:hasChild rdf:type owl:AsymmetricProperty ; owl:inverseOf :hasParent ;
  rdfs:subPropertyOf :ancestorOf .
:ancestorOf rdf:type owl:TransitiveProperty .
ex:Carmela :hasChild ex:Michael .
ex:Michael :hasDaughter ex:Mary .
ex:Mary a :Person .
:Person owl:equivalentClass
  [ owl:qualifiedCardinality 2 ;
    owl:onProperty :hasParent ;
    owl:onClass :Person ] .
```

```
ex:Michael :hasParent ex:Carmela .
ex:Michael :hasChild ex:Mary .
ex:Carmela :ancestorOf ex:Mary .
```

```
:Person rdfs:subClassOf [ owl:maxQualifiedCardinality 2 ;
   owl:onProperty :hasParent ; owl:onClass :Person ] .
:Person rdfs:subClassOf [ owl:maxQualifiedCardinality 3 ;
   owl:onProperty :hasParent ; owl:onClass :Person ] . ...
```

Materialisation:

Write down entailments

Entailments are infinite …

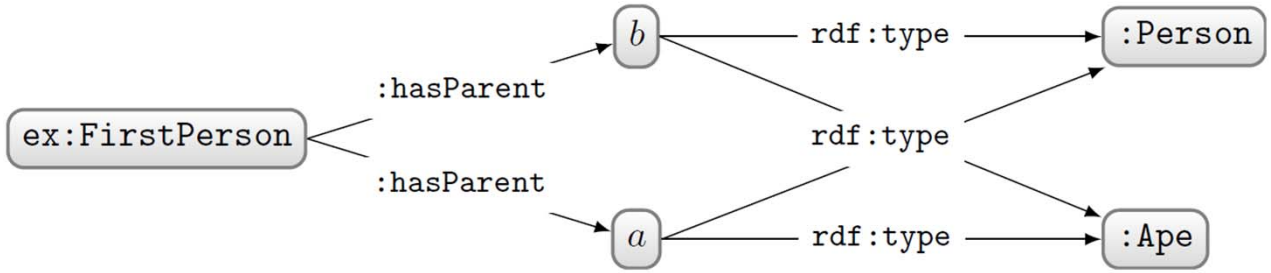
… which makes it tricky to write them all down …

Ontology Satisfiability:

Does *O* have a “model”?

```
:Person owl:equivalentClass
  [ owl:qualifiedCardinality 2 ; owl:onProperty :hasParent ;
    owl:onClass :Person ] .
:FirstPerson a :Person ,
  [ owl:qualifiedCardinality 2 ; owl:onProperty :hasParent ;
    owl:onClass :Ape ] .
```

So does *O* have a model?



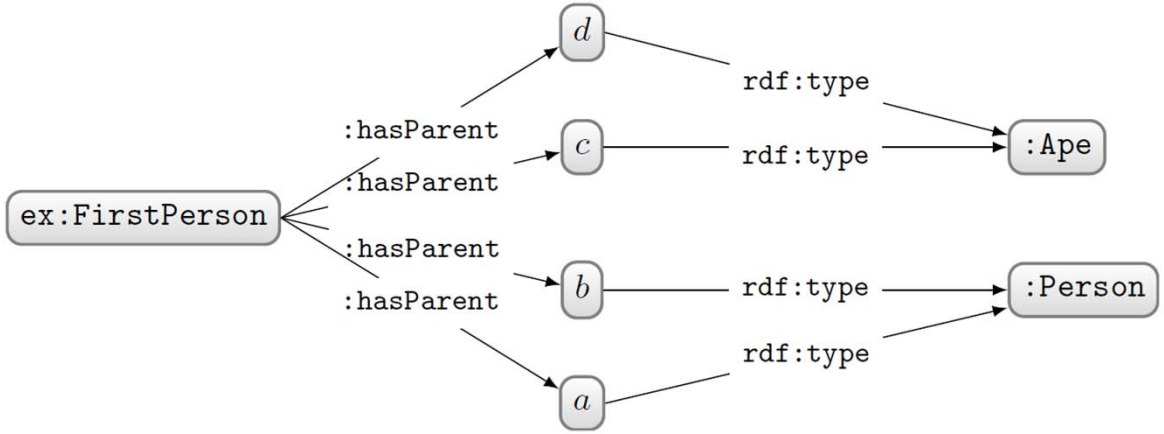
YES! Ontology *O* is **Satisfiable**!

Ontology Satisfiability:

Does *O* have a “model”?

```
:Person owl:equivalentClass
  [ owl:qualifiedCardinality 2 ; owl:onProperty :hasParent ;
    owl:onClass :Person ] .
:FirstPerson a :Person ,
  [ owl:qualifiedCardinality 2 ; owl:onProperty :hasParent ;
    owl:onClass :Ape ] .
:Ape owl:disjointWith :Person .
```

So does *O* have a model now?



YES! Ontology *O* is still **Satisfiable**!

Ontology Satisfiability:

Does *O* have a "model"?

```
:Person owl:equivalentClass
  [ owl:qualifiedCardinality 2 ; owl:onProperty :hasParent ;
    owl:onClass :Person ] .
:FirstPerson a :Person ,
  [ owl:qualifiedCardinality 2 ; owl:onProperty :hasParent ;
    owl:onClass :Ape ] .
:Ape owl:disjointWith :Person .
```

What more would we have to add to *O* to make it Unsatisfiable?

```
:Person rdfs:subClassOf
  [ owl:cardinality 2 ; owl:onProperty :hasParent ] .
```

OR

```
:Person owl:equivalentClass
  [ owl:allValuesFrom :Person ; owl:onProperty :hasParent ] .
```

OR

```
:FirstPerson a :Ape .
```

OR

```
:FirstPerson a owl:Nothing .
```

OR …

Ontology Satisfiability:

Does *O* have a “model”?

```
:Person owl:equivalentClass
  [ owl:qualifiedCardinality 2 ; owl:onProperty :hasParent ;
```

An unsatisfiable ontology cannot model any world!

It is inconsistent!

```
:FirstPerson a :Ape .
```

OR

```
:FirstPerson a owl:Nothing .
```

OR …

Entailment checking:

Does *O* entail *O′*?

```
:hasDaughter rdfs:subPropertyOf :hasChild .
:hasChild rdf:type owl:AsymmetricProperty ; owl:inverseOf :hasParent ;
  rdfs:subPropertyOf :ancestorOf .
:ancestorOf rdf:type owl:TransitiveProperty .
ex:Carmela :hasChild ex:Michael .
ex:Michael :hasDaughter ex:Mary .
```

```
ex:Michael :hasParent ex:Carmela .
ex:Michael :hasChild ex:Mary .
ex:Carmela :ancestorOf ex:Mary .
```

Alternatively: Are all models of *O* models of *O′* too?

REASONING …

How can we perform reasoning?

- Does Ontology O entail O'?

```
:hasDaughter rdfs:subPropertyOf :hasChild .
ex:Michael :hasDaughter ex:Mary .
```

```
ex:Michael :hasChild ex:Mary .
```

- Could instead ask: is "$O \cup \neg O'$" unsatisfiable?

```
:hasDaughter rdfs:subPropertyOf :hasChild .
ex:Michael :hasDaughter ex:Mary .
[] owl:sourceIndividual ex:Michael ;
   owl:assertionProperty :hasChild ;
   owl:targetIndividual ex:Mary .
```

Can reduce entailment to unsatisfiability!

So how do we test unsatisfiability then?

(Audrey was clearly not a Computer Scientist)

UNDECIDABILITY …

A simple Java program …

```
public class Collatz {
    public static void collatz(int n) {
        StdOut.print(n + " ");
        if (n == 1) return;
        else if (n % 2 == 0) collatz(n / 2);
        else collatz(3*n + 1);
    }

    public static void main(String[] args) {
        int N = Integer.parseInt(args[0]);
        collatz(N);
        StdOut.println();
    }

}
```

In: 6

```
3
10
5
16
8
4
2
1 (end)
```

Does this Java program terminate on all possible (positive) inputs?

A simple Java program …

```
public class Collatz {
    public static void collatz(int n) {
        StdOut.print(n + " ");
        if (n == 1) return;
        else if (n % 2 == 0) collatz(n / 2);
        else collatz(3*n + 1);
    }

    public static void main(String[] args) {
        int N = Integer.parseInt(args[0]);
        collatz(N);
        StdOut.println();
    }

}
```

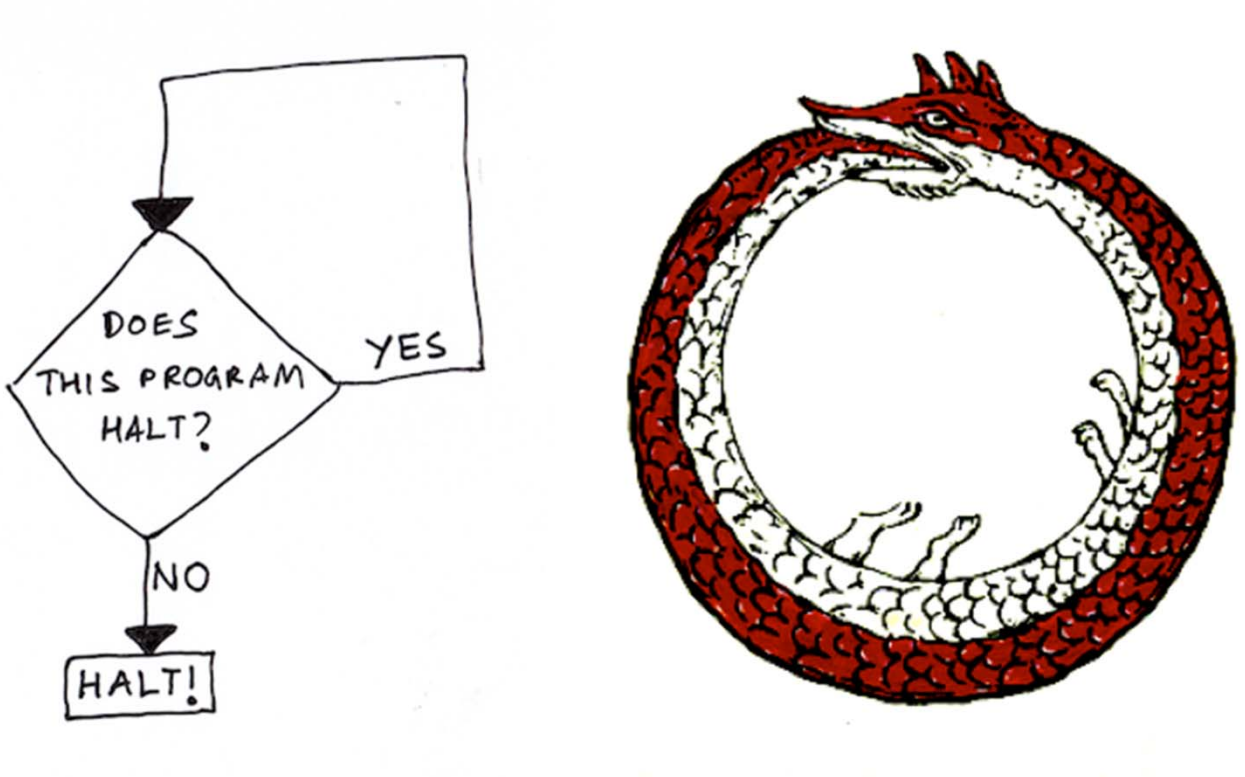
- **Collatz conjecture**: an unsolved problem in mathematics
- If we knew that this program terminates or does not terminate on all natural numbers (not just `ints`), this problem would be solved!

Halting Problem

- Input: a program and an input to that program
- Output:
  - `true` if the program halts on that input
  - `false` otherwise

- UNDECIDABLE: a general algorithm to solve the Halting Problem does not exist!
  - It will not halt for all program–input pairs!
  - It may halt for some program–input pairs

A quick *sketch* of Halting Problem proof



Problem: Consecutive '1's in π

- Input: A natural number *n*
- Output:
  - `true` if π contains *n* consecutive '1's
  - `false` otherwise

Is this problem **DECIDABLE** or **UNDECIDABLE**?

… i.e., does there exist a program that halts (with the correct answer) for all *n*?

What if we knew the maximum sequence of consecutive '1's in π?

```
if (n ≤ MAX) return true; else return false;
```

• there must exist a `MAX` sequence of consecutive '1's in π (even if it's ∞)

∴ there must exist a correct program that halts (even if we don't know its details)

∴ problem is **DECIDABLE**!

Problem: Collatz Halting Problem

- Input: [none]
- Output:
  - `true` if Collatz program halts on all inputs
  - `false` otherwise

Is this problem **DECIDABLE** or **UNDECIDABLE**?

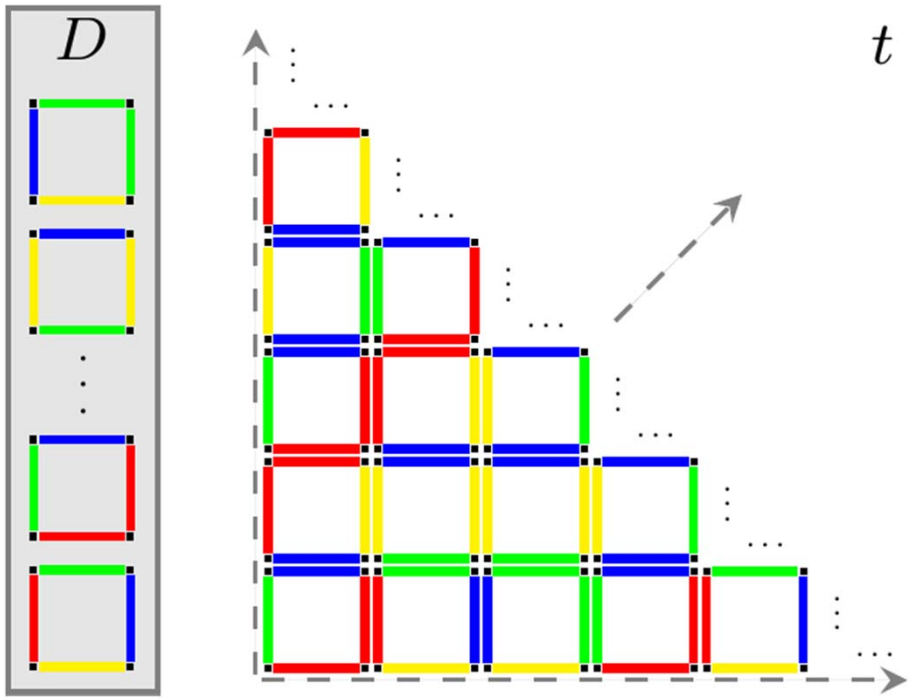
(P1) `return true;`

(P2) `return false;`

• either **(P1)** or **(P2)** must be correct
∴ there must exist a correct program that halts (even if we don't know it)
∴ problem is **DECIDABLE**!

Halting Problem **UNDECIDABLE** in the general case
(for all programs and inputs)

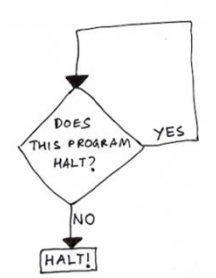
Domino Tiling Problem

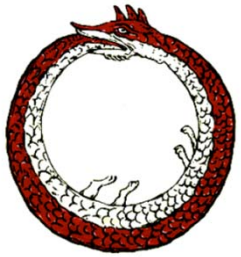


Is this problem **DECIDABLE** or **UNDECIDABLE**?

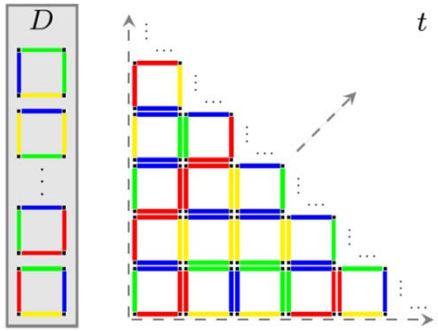
- Input: A set of Dominos (like *D*)
- Output:
  - `true` if there exists a valid infinite tiling (like *t*)
  - `false` otherwise

Can reduce from Halting to Tiling





Reduction



It has been shown that there exists a program that can reduce any Halting problem instance into a Domino Tiling problem instance

Now is the Domino Tiling problem **DECIDABLE** or **UNDECIDABLE**?

```
halts(p,in) {
        D = reduce(p,in);
        return hasTiling(D);
}
```

- `reduce(p,in)` is **DECIDABLE**

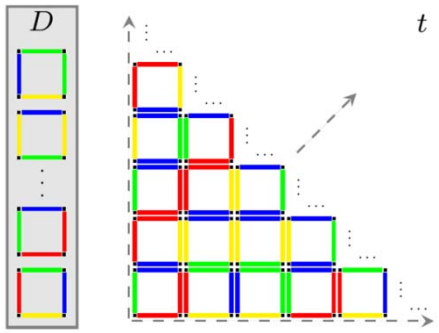
∴ if `hasTiling(D)` were **DECIDABLE**, then `halts(p,in)` would be **DECIDABLE**

- but `halts(p,in)` is **UNDECIDABLE**

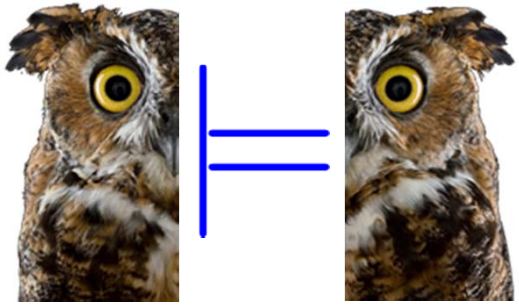
∴ `hasTiling(D)` must be **UNDECIDABLE**!

*If we have a decidable reduction from an **UNDECIDABLE** problem A to another problem B, then B must be **UNDECIDABLE***

Reduce from Tiling to OWL entailment?



Reduction



Does D have an infinite tiling?

Does OWL ontology *O* entail *O'*?

How can we encode a Domino Tiling question into an OWL ontology entailment question?

If we could do this, we could save ourselves trying to code a program to implement OWL entailment since we would know it was **UNDECIDABLE**

Some Description Logic symbols

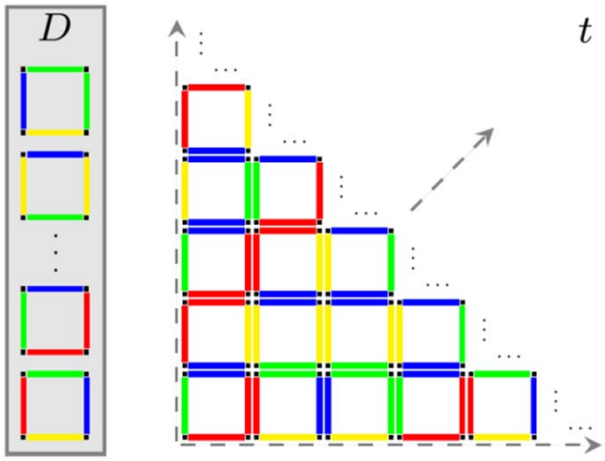
- ⊑: sub-class/-property
- ≡: equivalent class/property
- ⊔: union
- ⊓: intersection
- ⊤: top (class of everything)
- ⊥: bottom (empty class)
- ∃: exists (`someValuesFrom`/`hasValue`)
- ∀: for all (`allValuesFrom`)
- ¬: not (complement, negation)
- ⁻ (superscript minus): inverse property
- {}: enumeration (`owl:oneOf`)
- Self, Trans, Dom, etc.: where symbols not available
- ∘: property chain
- C(x): class membership
- P(x,y): a triple `(x,P,y)`

Can reduce from OWL entailment to Tiling

1. Each tile must be a domino
   - Define dominos as a class *D*, a union of classes for each domino type:

$$D \equiv D_1 \sqcup D_2 \sqcup \ldots \sqcup D_{k-1} \sqcup D_k$$

Now what else do we need to encode?



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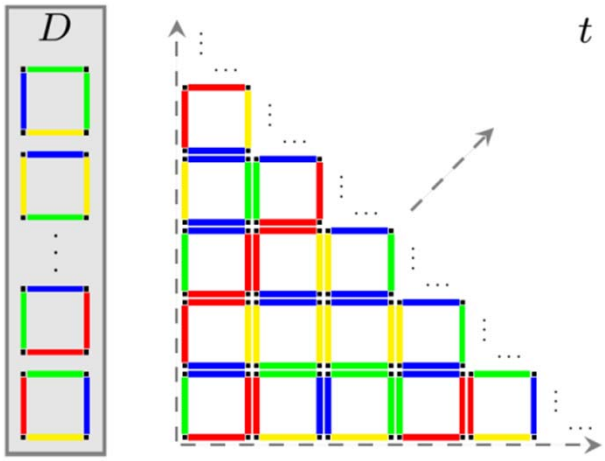
2. Each tile can only be one domino type

How can we encode this in OWL?



Can reduce from OWL entailment to Tiling

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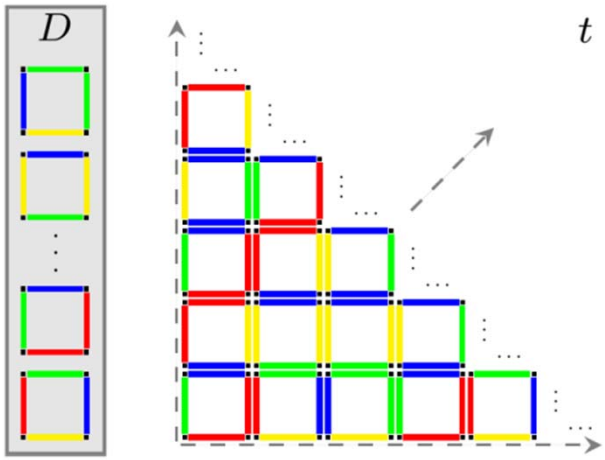
$$D \equiv D_1 \sqcup D_2 \sqcup \ldots \sqcup D_{k-1} \sqcup D_k$$

2. Each tile can only be one domino type
   - Define dominos types as pairwise disjoint:

$$D_i \sqcap D_j \sqsubseteq \bot (\text{for } 1 \leq i < j \leq k)$$

What else do we need to encode?



Can reduce from OWL entailment to Tiling

1. Each tile must be a domino
   - Define dominos as a class *D*, a union of classes for each domino type:

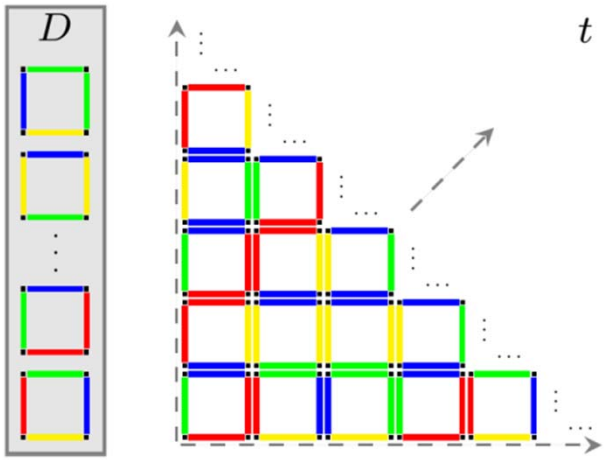
$$D \equiv D_1 \sqcup D_2 \sqcup \ldots \sqcup D_{k-1} \sqcup D_k$$

2. Each tile can only be one domino type
   - Define dominos types as pairwise disjoint:

$$D_i \sqcap D_j \sqsubseteq \bot (\text{for } 1 \leq i < j \leq k)$$

3. Each tile must have a tile to the right and above

How can we encode this?



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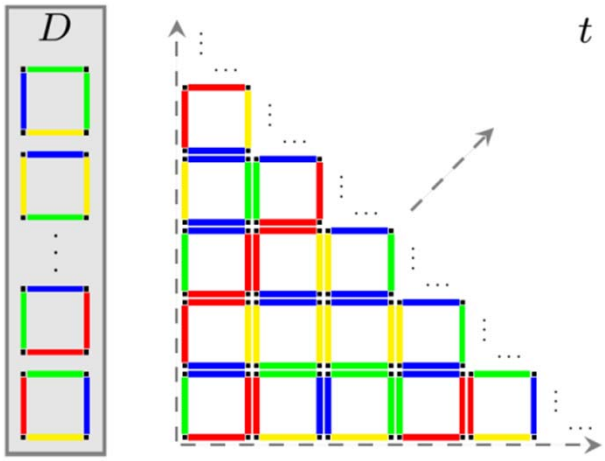
3. Each tile must have a tile to the right and above
   - Define that a domino has some values from domino for *r*ight/*a*bove:

$$D \sqsubseteq (\exists r.D) \sqcap (\exists a.D)$$

Are we there yet?



Can reduce from OWL entailment to Tiling

4. Tiles to the right and tiles above must match colour
   - Define that a domino has all values from matching tiles right/above:

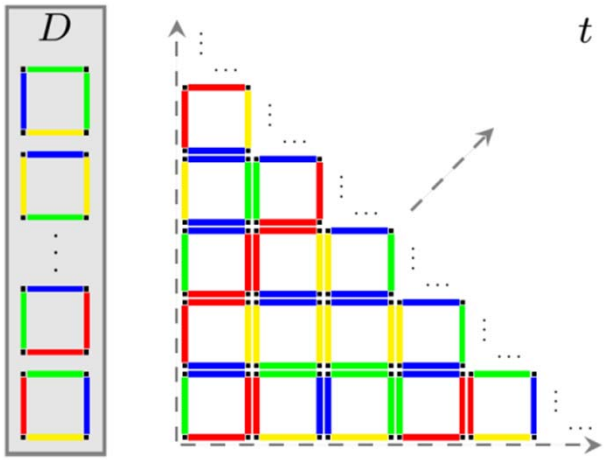
$$D_1 \sqsubseteq \forall r.(\bigsqcup_{D' \in R(D_1)} D') \sqcap \forall a.(\bigsqcup_{D' \in A(D_1)} D')$$

$$\dots$$

$$D_k \sqsubseteq \forall r.(\bigsqcup_{D' \in R(D_k)} D') \sqcap \forall a.(\bigsqcup_{D' \in A(D_k)} D')$$

Where:
$R(D_i)$ denotes all domino types that can be to the right of D_i
$A(D_i)$ denotes all domino types that can be above D_i

Are we there yet?

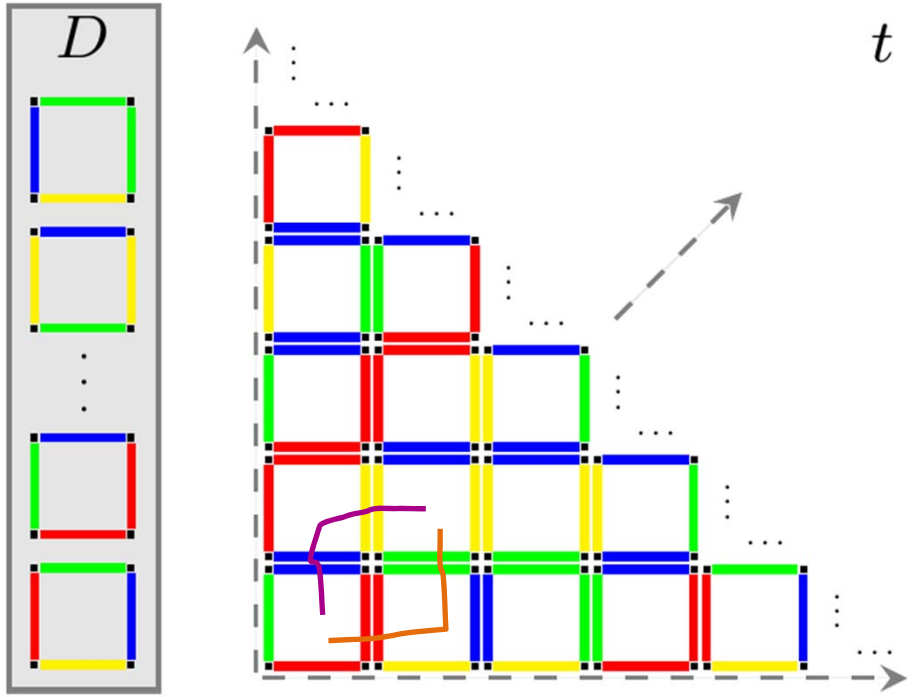


What condition are we missing?

- So far we could still have trees as models
- Need to state:

  Tile above and to the right = Tile to the right and above

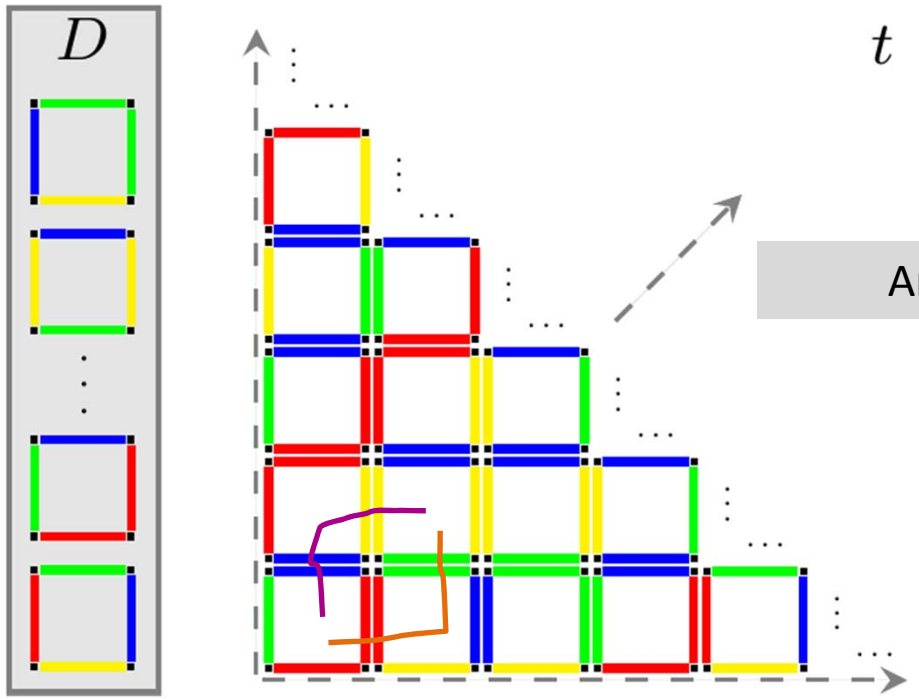
How can we encode this?



Can reduce from OWL entailment to Tiling

5. Tile right then above = Tile above then right
   - Define *d*iagonal tile using two property chains (above-right/right-above)
   - Declare functional (a tile can only have one such diagonal tile)

$$d \sqsubseteq a \circ r, \quad d \sqsubseteq r \circ a, \quad \mathsf{Func}(d)$$



Are we there yet?

What's the entailment question?

$$D \equiv D_1 \sqcup D_2 \sqcup \ldots \sqcup D_{k-1} \sqcup D_k$$
$$D_i \sqcap D_j \sqsubseteq \bot (\text{for } 1 \leq i < j \leq k)$$
$$D \sqsubseteq (\exists r.D) \sqcap (\exists a.D)$$
$$D_1 \sqsubseteq \forall r.(\bigsqcup_{D' \in R(D_1)} D') \sqcap \forall a.(\bigsqcup_{D' \in A(D_1)} D')$$
$$\ldots$$
$$D_k \sqsubseteq \forall r.(\bigsqcup_{D' \in R(D_k)} D') \sqcap \forall a.(\bigsqcup_{D' \in A(D_k)} D')$$
$$d \sqsubseteq a \circ r, \quad d \sqsubseteq r \circ a, \quad \mathsf{Func}(d)$$

???

What should we put in $\mathcal{O}'$?

Goal: Ontology $\mathcal{O}$ entails $\mathcal{O}'$ if and only if D has an infinite tiling

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$$d \sqsubseteq a \circ r, \quad d \sqsubseteq r \circ a, \quad \mathsf{Func}(d)$$

$$D \equiv \bot$$

Goal: Ontology $\mathcal{O}$ entails $\mathcal{O}'$ if and only if *D* has an infinite tiling

If *D* has <u>any</u> member (a "tile"), it must have an infinite tiling!

If *D* has <u>no</u> member, it must not have an infinite tiling.

Could also use satisfiability

$$D \equiv D_1 \sqcup D_2 \sqcup \ldots \sqcup D_{k-1} \sqcup D_k$$

$$D_i \sqcap D_j \sqsubseteq \bot (\text{for } 1 \leq i < j \leq k)$$

$$D \sqsubseteq (\exists r.D) \sqcap (\exists a.D)$$

$$D_1 \sqsubseteq \forall r.(\bigsqcup_{D' \in R(D_1)} D') \sqcap \forall a.(\bigsqcup_{D' \in A(D_1)} D')$$

$$\ldots$$

$$D_k \sqsubseteq \forall r.(\bigsqcup_{D' \in R(D_k)} D') \sqcap \forall a.(\bigsqcup_{D' \in A(D_k)} D')$$

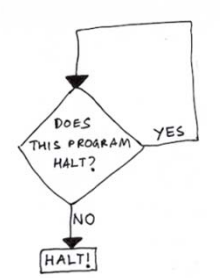
$$d \sqsubseteq a \circ r, \quad d \sqsubseteq r \circ a, \quad \mathsf{Func}(d)$$

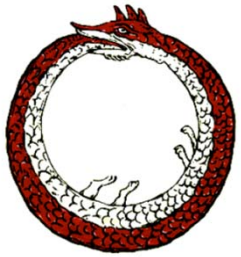
$$D(x)$$

Ontology *O* is satisfiable if and only if *D* has an infinite tiling

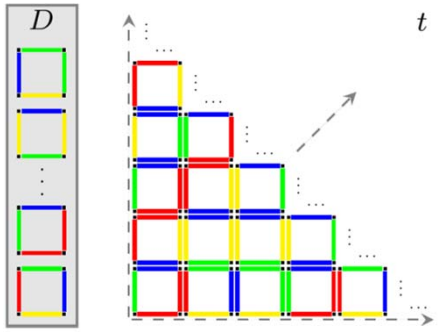
Here, *x* is an arbitrary fresh term

OWL satisfiability/entailment is powerful





Reduction



OWL satisfiability/entailment also **undecidable**!

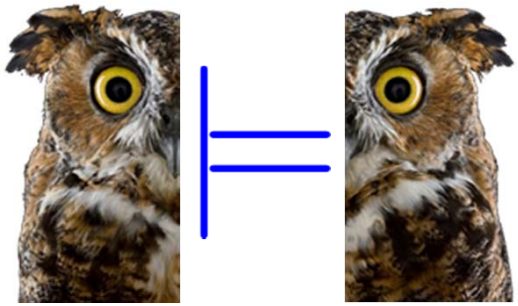
Otherwise could be used to solve Domino Tiling problem …

… and the Halting problem …

… and (given enough time), the Collatz conjecture …

… and a bunch of other stuff

Reduction



PRACTICAL REASONING

Options …

Well great. What are we supposed to do now?

- Accept incomplete reasoners that halt
  - You may not get all the entailments … so what entailments do you get?
- Accept complete reasoners that may not halt
  - Java is a language that lets you write programmes that may not halt
- Restrict OWL so we can't solve Halting/Tiling
  - Main problem tackled in Description Logics field: find decidable sublanguages of OWL without turning off too many features (and allowing efficient algorithms)

More next week …

RECAP


RDFS
L

OWL Definitions

- Models
  - A world that the ontology could be true for
    - … and a mapping from the terms of the ontology to that world
  - Not necessarily the real world (just a consistent world)
  - Can be much larger than the ontology describes
  - The more detailed the ontology, the fewer its models

- Entailment
  - An ontology *O* entails another ontology *O'* if any model of *O* is a model of *O'*
    - … in which case *O'* adds no new information to *O*
    - *… in which case O' follow from O*

OWL Reasoning Tasks

- Materialisation
  - Write down all the entailments from *O*′
    - *Unfortunately, they are infinite*
- Satisfiability
  - Does an ontology have any model?
    - If not, it is inconsistent/unsatisfiable
    - *Unfortunately, satisfiability is undecidable*
- Entailment (checking)
  - Does an ontology *O* entail another ontology *O*′?
    - If so, *O*′ follows as a consequence of *O*
    - Can be reduced to satisfiability
    - *Unfortunately, entailment checking is also undecidable*

Questions?