CC5212-1

Procesamiento Masivo de Datos

Otoño 2018

Lecture 7

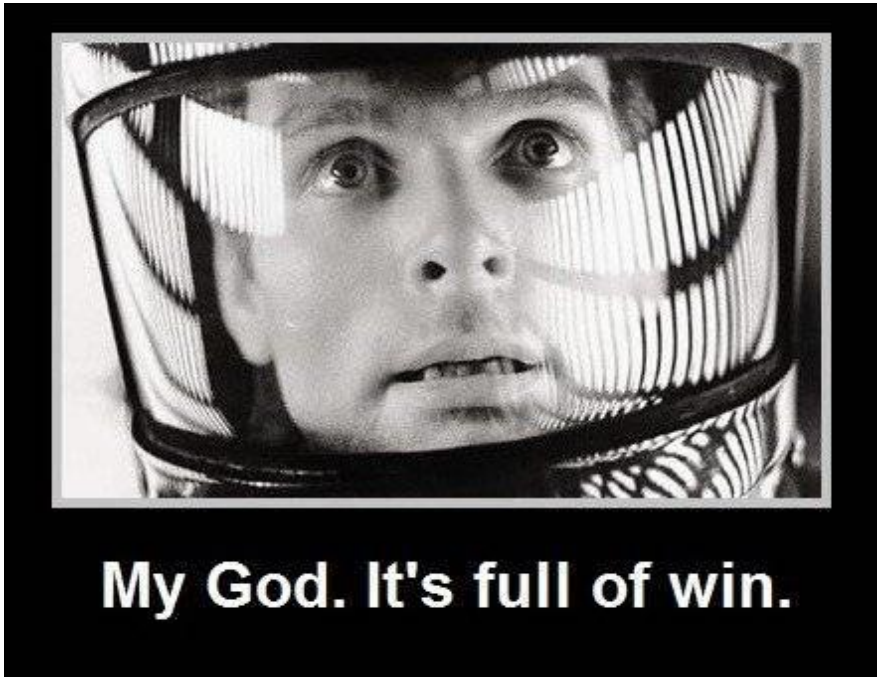
Information Retrieval: Ranking

Aidan Hogan

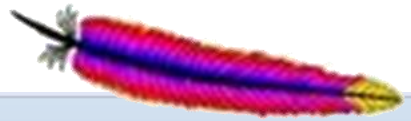
aidhog@gmail.com

Apache Lucene

- Inverted Index
  - They built one so you don't have to!
  - Open Source in Java



Apache Lucene



Tasks | Console

SearchWikiIndex [Java Application] C:\Program Files\Java\jre1.8.0_65\bin\javaw.exe (03-05-2017 12:45:22 a. m.)

```
Opening directory at  lucene
Enter a keyword search phrase:
obama
Running query: obama
Parsed query: TITLE:obam^5.0 ABSTRACT:obam
Matching documents: 255
Showing top 10 results
1       http://es.wikipedia.org/wiki/Obama_Republican   Obama Republican
2       http://es.wikipedia.org/wiki/Obama_(Fukui)      Obama (Fukui)
3       http://es.wikipedia.org/wiki/Republicanos_por_Obama     Republicanos por Obama
4       http://es.wikipedia.org/wiki/Engonga_Obame      Engonga Obame
5       http://es.wikipedia.org/wiki/Barack_Obama       Barack Obama
6       http://es.wikipedia.org/wiki/Michelle_Obama     Michelle Obama
7       http://es.wikipedia.org/wiki/Cartel_%22Hope%22_de_Obama Cartel "Hope" de Obama
8       http://es.wikipedia.org/wiki/Transición_presidencial_de_Barack_Obama    Transición presidencial de Barack Obama
9       http://es.wikipedia.org/wiki/Por_qué_Obama_ganará_en_2008_y_en_2012     Por qué Obama ganará en 2008 y en 2012
10      http://es.wikipedia.org/wiki/Ricardo_Mangue_Obama_Nfubea        Ricardo Mangue Obama Nfubea
```

Information Retrieval: Ranking

How Does Google Get Such Good Results?

Google

obama

All News Images Videos More Settings Tools

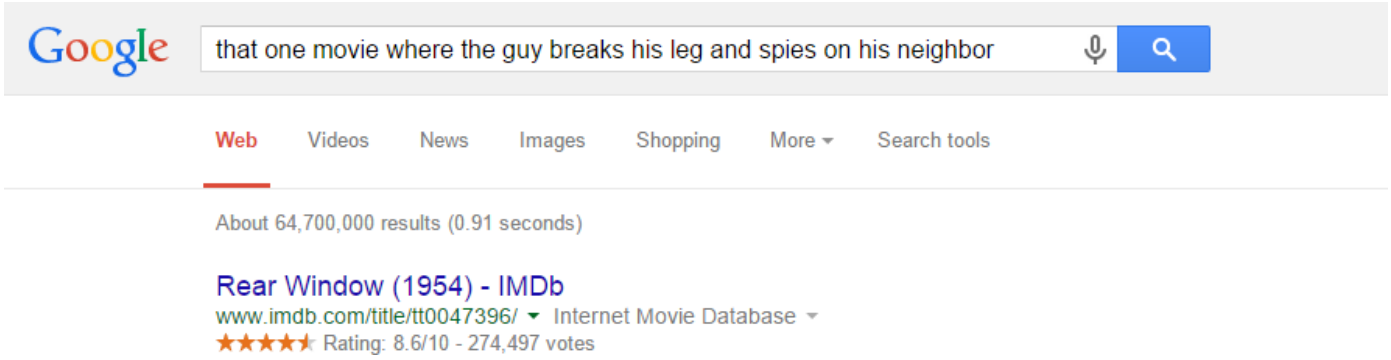
About 462,000,000 results (0.71 seconds)

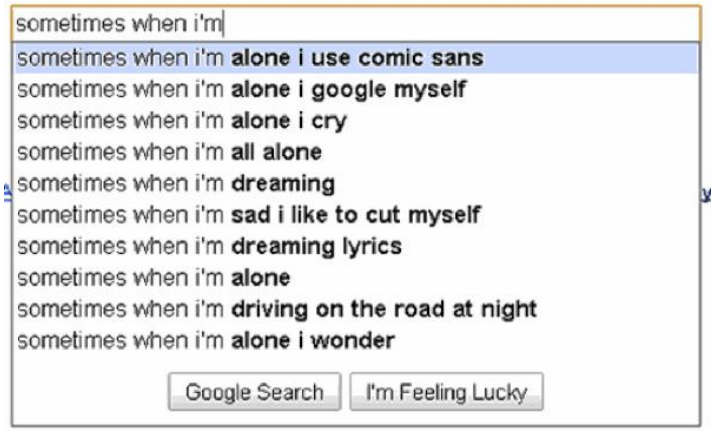
Barack Obama (@BarackObama) · Twitter
https://twitter.com/BarackObama
Well said, Jimmy. That's exactly why we fought so hard for the ACA, and why we need to protect it for kids like Billy. And congratulations! twitter.com/jimmykimmel...
11 hours ago · Twitter

The Office of Barack and Michelle Obama
https://www.barackobama.com/
Welcome to the Office of Barack and Michelle **Obama**. We Love You Back. Play video. The Office of Barack and Michelle **Obama**. © 2017 | Legal & Privacy.

Barack Obama - Wikipedia
https://en.wikipedia.org/wiki/Barack_Obama
Barack Hussein **Obama** II is an American politician who served as the 44th President of the United States from 2009 to 2017. He is the first African American to ...

How does Google Get Such Good Results?





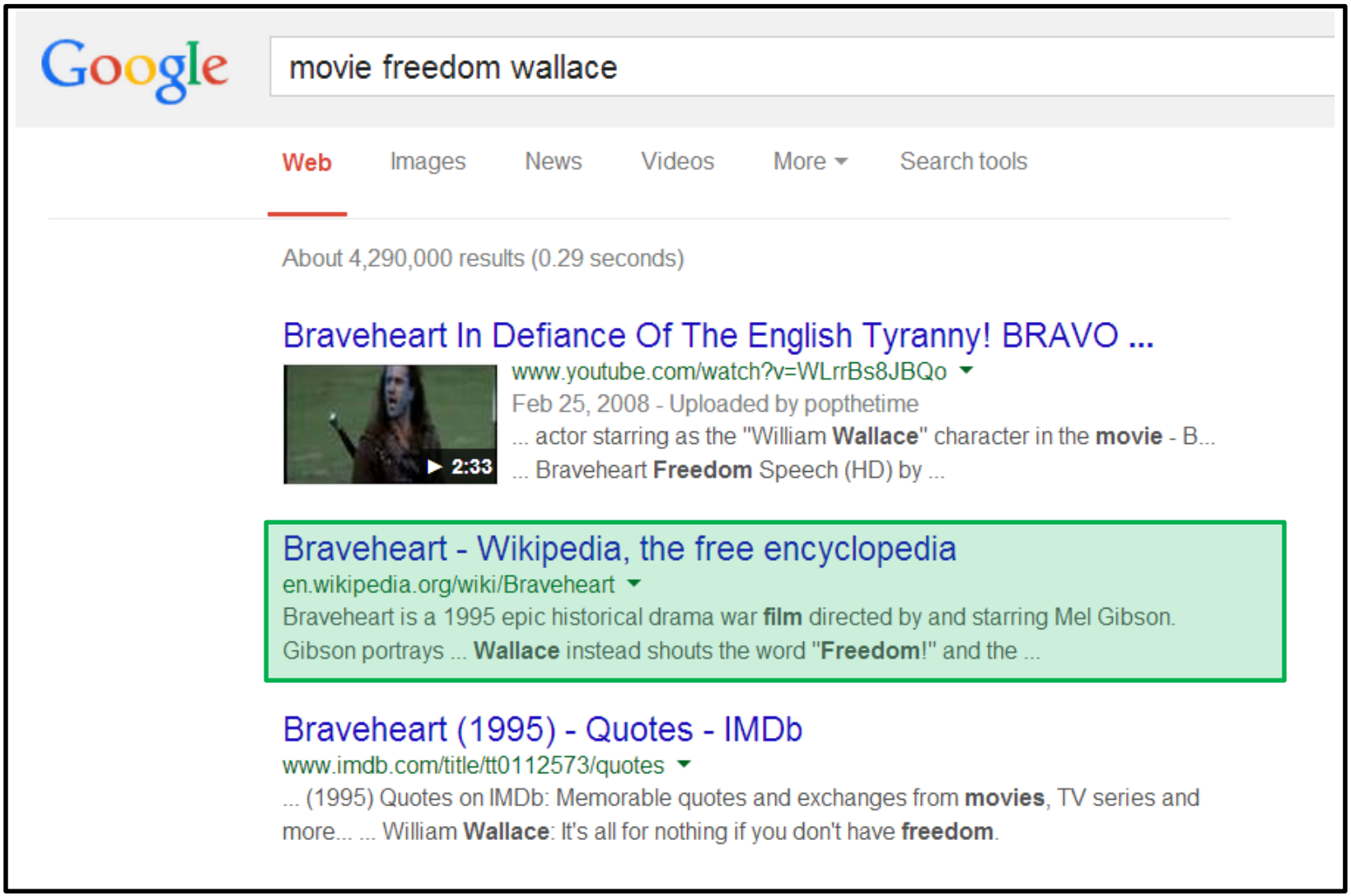
Two Sides to Ranking: Relevance

Two Sides to Ranking: Importance

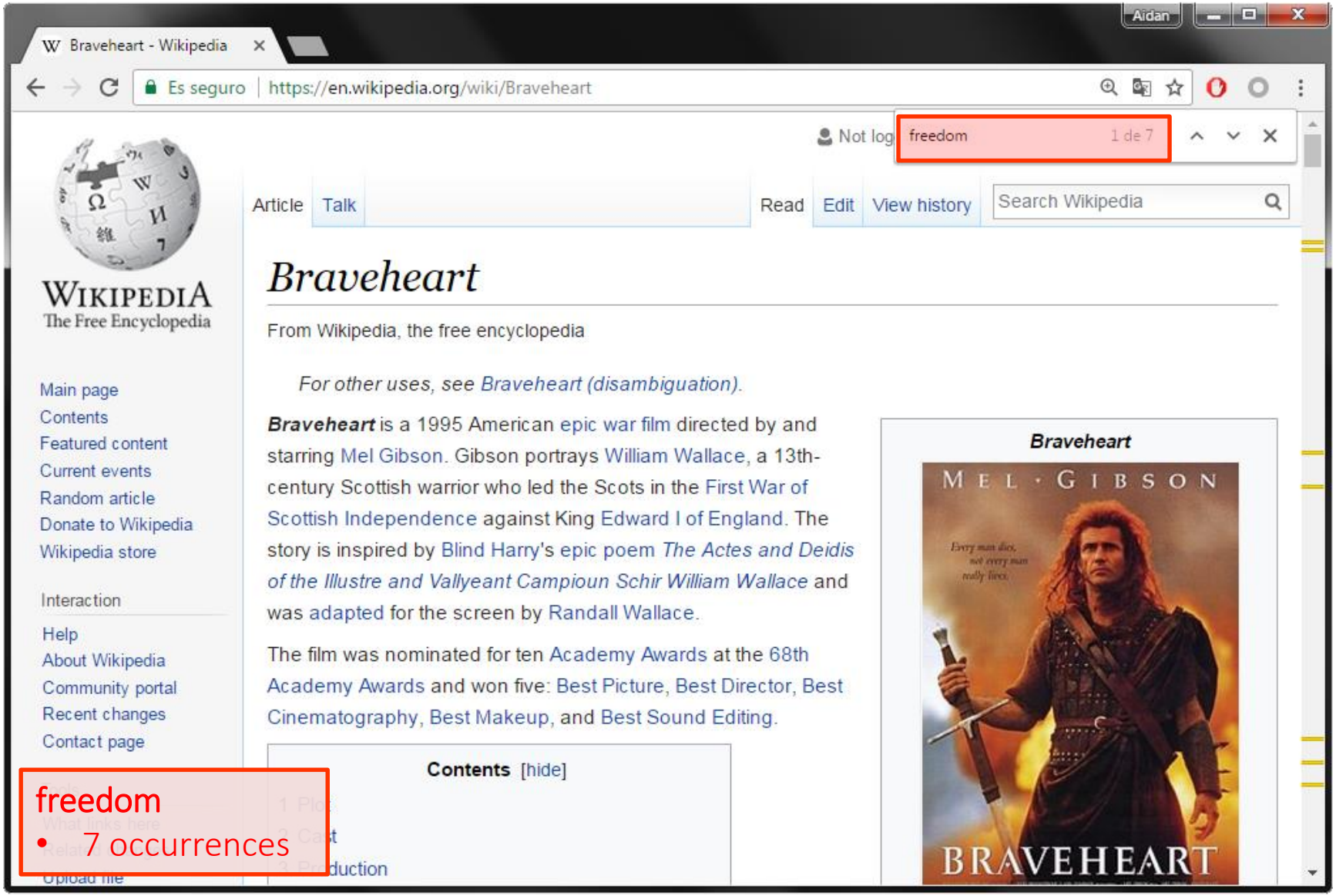
Ranking: Relevance

Example Query

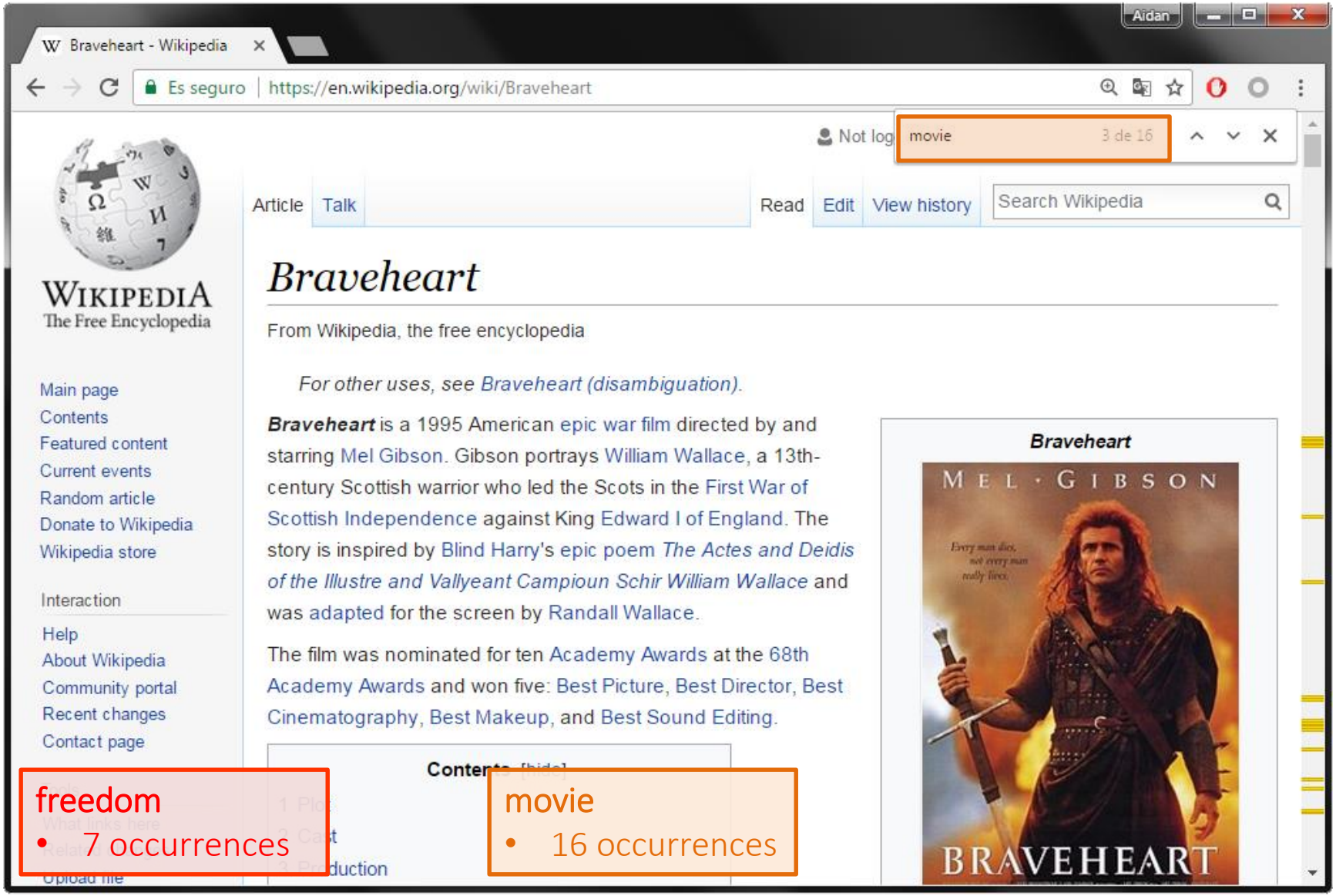
Which of these three keyword terms is most “important”?



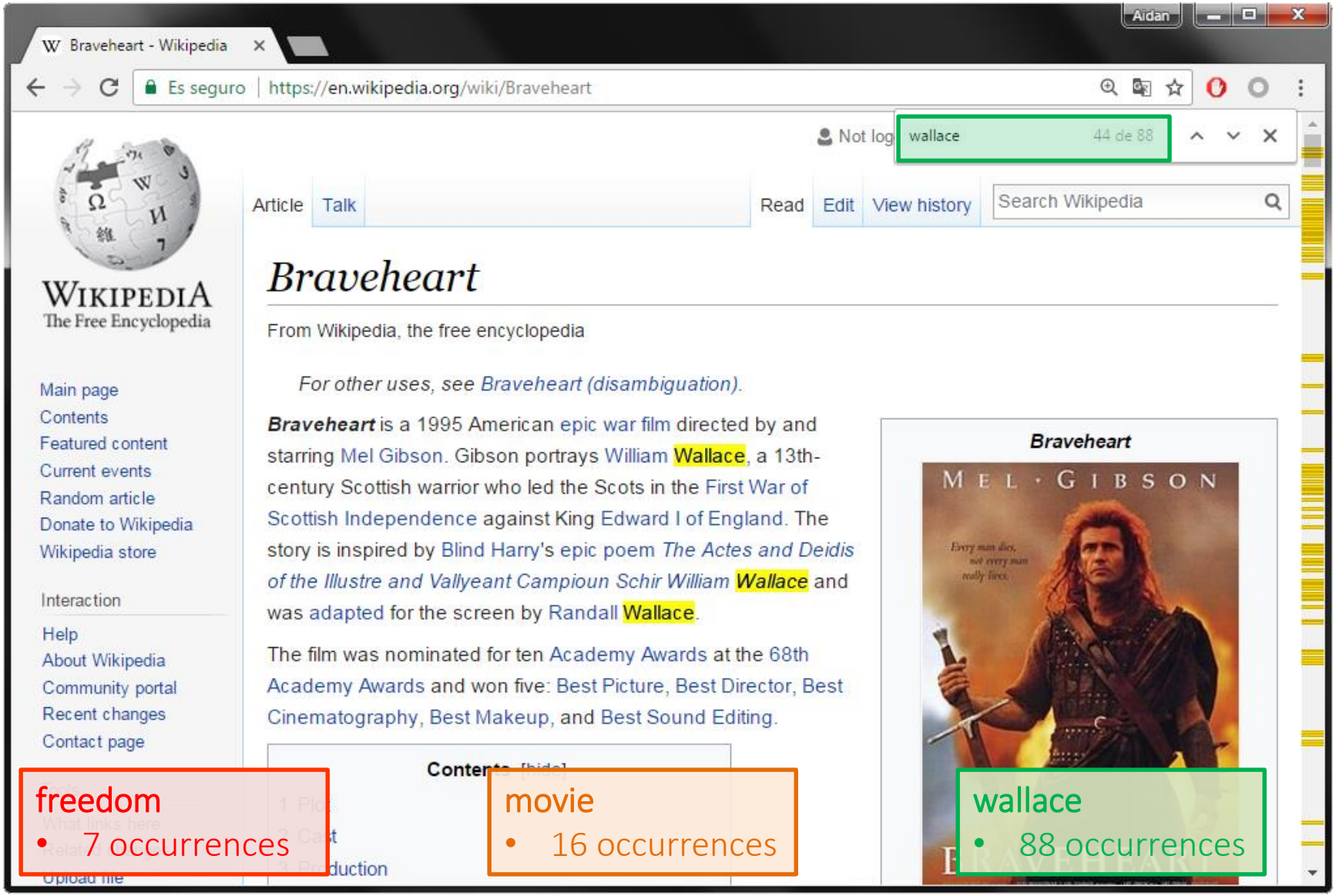
Matches in a Document



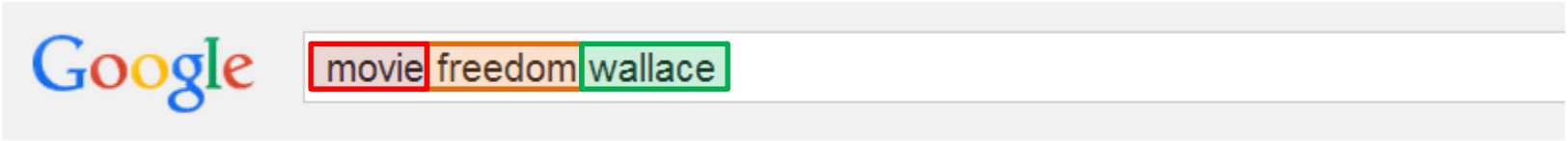
Matches in a Document

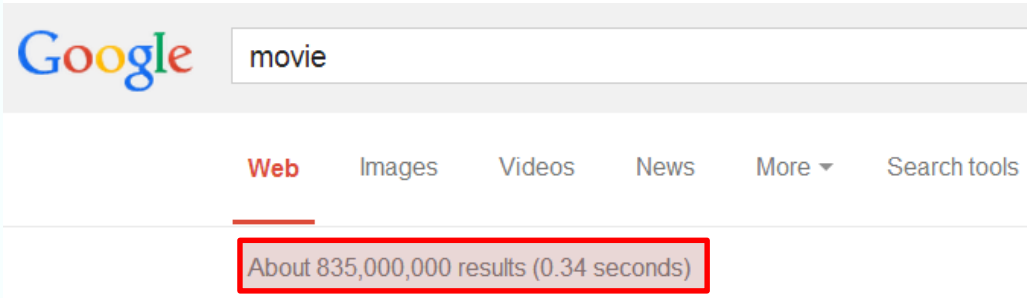


Matches in a Document

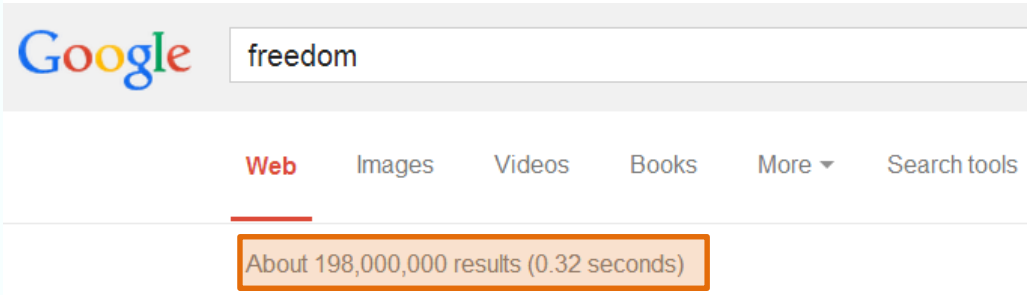


Usefulness of Words

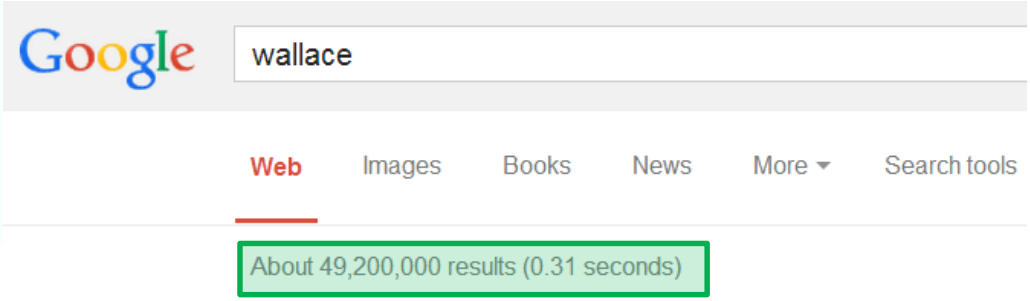




movie
- occurs very frequently



freedom
- occurs frequently



wallace
- occurs occassionally

Estimating Relevance

- Rare words more important than common words
  - wallace (49M) more important than freedom (198M) more important than movie (835M)
- Words occurring more frequently in a document indicate higher relevance
  - wallace (88) more matches than movie (16) more matches than freedom (7)

Relevance Measure: TF–IDF

- TF: Term Frequency
  - Measures occurrences of a term in a document
  - $\mathrm{tf}(t, d)$ … various options
    - Raw count of occurrences

$$\mathrm{tf}(t, d) = \mathrm{count}(t, d)$$

    - Logarithmically scaled

$$\mathrm{tf}(t, d) = \log(\mathrm{count}(t, d) + 1)$$

    - Normalised by document length

$$\mathrm{tf}(t, d) = \frac{\mathrm{count}(t,d)}{\sum_{t' \in d} \mathrm{count}(t',d)}$$

$$\mathrm{tf}(t, d) = \frac{\mathrm{count}(t,d)}{\max_{t' \in d} \mathrm{count}(t',d)}$$

    - A combination / something else ☺

Relevance Measure: TF–IDF

- IDF: Inverse Document Frequency
  - How common a term is across **all** documents
  - $\mathrm{idf}(t, D)$ …
    - Logarithmically scaled document occurrences

$$\mathrm{idf}(t, D) = \log\left(\frac{|D|+1}{|\{d \in D : t \in d\}|+1}\right)$$

    - Note: The more rare, the larger the value

Relevance Measure: TF–IDF

- TF–IDF: Combine Term Frequency and Inverse Document Frequency:

$$\text{tf-idf}(t, d) = \text{tf}(t, d) \times \text{idf}(t, D)$$

- Score for a query
  - Let query $q = (t_1, \ldots, t_n)$
  - Score for a query: $\text{score}(q, d) = \sum_{t \in q} \text{tf-idf}(t, d)$

  (There are other possibilities)

Relevance Measure: TF–IDF

Term Frequency

$$\mathrm{tf}(t,d) = \mathrm{count}(t,d)$$

Inverse Document Frequency

$$\mathrm{idf}(t,D) = \log_2\left(\frac{|D|}{|\{d \in D : t \in d\}|+1}\right)$$

$$\text{tf-idf}(t,d) = \mathrm{tf}(t,d) \times \mathrm{idf}(t,D)$$

t	$\mathrm{tf}(t,d)$
movie	16
freedom	7
wallace	43

Relevance Measure: TF–IDF

Term Frequency

$$\mathrm{tf}(t,d) = \mathrm{count}(t,d)$$

Inverse Document Frequency

$$\mathrm{idf}(t,D) = \log_2\left(\frac{|D|}{|\{d \in D : t \in d\}|+1}\right)$$

$$\text{tf-idf}(t,d) = \mathrm{tf}(t,d) \times \mathrm{idf}(t,D)$$

t	$\mathrm{tf}(t,d)$	$\lvert\{d \in D : t \in d\}\rvert$
movie	16	835,000,000
freedom	7	198,000,000
wallace	43	49,200,000

Relevance Measure: TF–IDF

Term Frequency

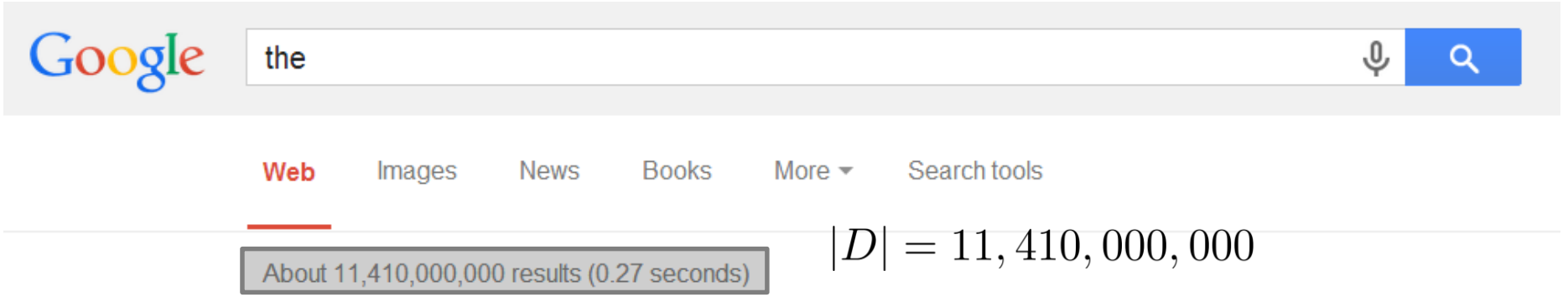
$$\mathrm{tf}(t,d) = \mathrm{count}(t,d)$$

Inverse Document Frequency

$$\mathrm{idf}(t,D) = \log_2\left(\frac{|D|}{|\{d \in D : t \in d\}|+1}\right)$$

$$\text{tf-idf}(t,d) = \mathrm{tf}(t,d) \times \mathrm{idf}(t,D)$$

t	$\mathrm{tf}(t,d)$	$\lvert\{d \in D : t \in d\}\rvert$	$\frac{\lvert D\rvert}{\lvert\{d \in D : t \in d\}\rvert+1}$
movie	16	835,000,000	
freedom	7	198,000,000	
wallace	43	49,200,000	



$$|D| = 11,410,000,000$$

Relevance Measure: TF–IDF

Term Frequency

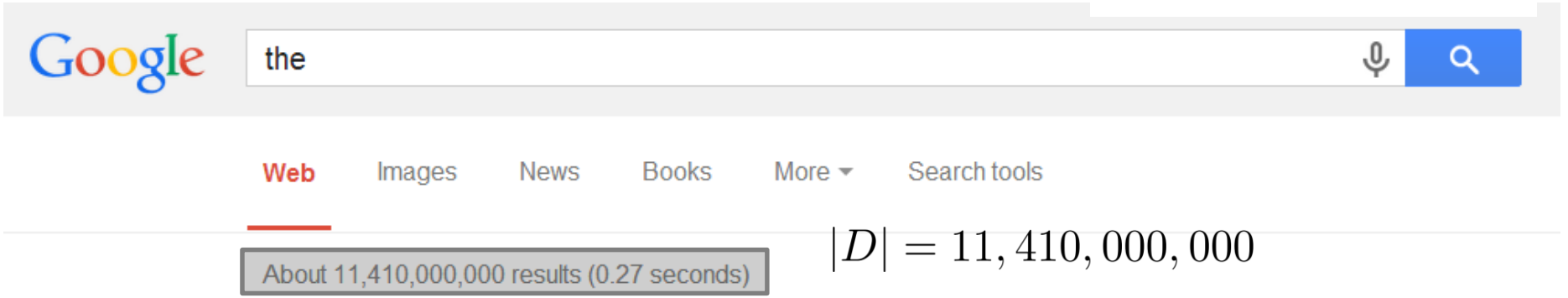
$$\mathrm{tf}(t,d) = \mathrm{count}(t,d)$$

Inverse Document Frequency

$$\mathrm{idf}(t,D) = \log_2\left(\frac{|D|}{|\{d \in D : t \in d\}|+1}\right)$$

$$\text{tf-idf}(t,d) = \mathrm{tf}(t,d) \times \mathrm{idf}(t,D)$$

t	$\mathrm{tf}(t,d)$	$\lvert\{d \in D : t \in d\}\rvert$	$\frac{\lvert D\rvert}{\lvert\{d \in D : t \in d\}\rvert+1}$
movie	16	835,000,000	13.66
freedom	7	198,000,000	57.62
wallace	43	49,200,000	231.91



$$|D| = 11,410,000,000$$

Relevance Measure: TF–IDF

Term Frequency

$$\mathrm{tf}(t,d) = \mathrm{count}(t,d)$$

Inverse Document Frequency

$$\mathrm{idf}(t,D) = \log_2\left(\frac{|D|}{|\{d \in D : t \in d\}|+1}\right)$$

$$\text{tf-idf}(t,d) = \mathrm{tf}(t,d) \times \mathrm{idf}(t,D)$$

t	$\mathrm{tf}(t,d)$	$\lvert\{d \in D : t \in d\}\rvert$	$\frac{\lvert D\rvert}{\lvert\{d \in D : t \in d\}\rvert+1}$	$\mathrm{idf}(t,D)$
movie	16	835,000,000	13.66	3.77
freedom	7	198,000,000	57.62	5.84
wallace	43	49,200,000	231.91	7.85

Relevance Measure: TF–IDF

Term Frequency

$$\mathrm{tf}(t,d) = \mathrm{count}(t,d)$$

Inverse Document Frequency

$$\mathrm{idf}(t,D) = \log_2\left(\frac{|D|}{|\{d \in D : t \in d\}|+1}\right)$$

$$\text{tf-idf}(t,d) = \mathrm{tf}(t,d) \times \mathrm{idf}(t,D)$$

t	$\mathrm{tf}(t,d)$	$\lvert\{d \in D : t \in d\}\rvert$	$\frac{\lvert D\rvert}{\lvert\{d \in D : t \in d\}\rvert+1}$	$\mathrm{idf}(t,D)$	$\text{tf-idf}(t,d)$
movie	16	835,000,000	13.66	3.77	60.36
freedom	7	198,000,000	57.62	5.84	40.94
wallace	43	49,200,000	231.91	7.85	337.87

Relevance Measure: TF–IDF

Term Frequency

$$\mathrm{tf}(t,d) = \mathrm{count}(t,d)$$

Inverse Document Frequency

$$\mathrm{idf}(t,D) = \log_2\left(\frac{|D|}{|\{d \in D : t \in d\}|+1}\right)$$

$$\text{tf-idf}(t,d) = \mathrm{tf}(t,d) \times \mathrm{idf}(t,D)$$

t	$\mathrm{tf}(t,d)$	$\lvert\{d \in D : t \in d\}\rvert$	$\frac{\lvert D\rvert}{\lvert\{d \in D : t \in d\}\rvert+1}$	$\mathrm{idf}(t,D)$	$\text{tf-idf}(t,d)$
movie	16	835,000,000	13.66	3.77	60.36
freedom	7	198,000,000	57.62	5.84	40.94
wallace	43	49,200,000	231.91	7.85	337.87

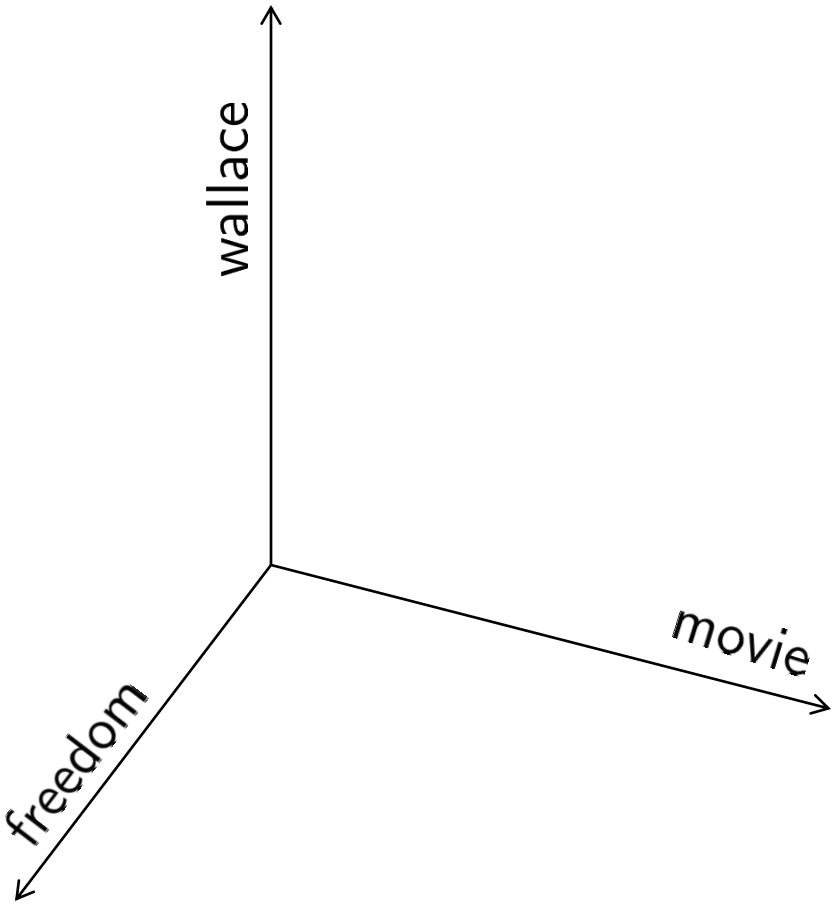
$$\mathrm{score}(q,d) = \sum_{t \in q} \text{tf-idf}(t,d)$$

$$\mathrm{score}\big((\text{movie}, \text{freedom}, \text{wallace}), \text{http://en.wikipedia.org/Braveheart}\big) \approx 439.17$$

Vector Space Model (a mention)

t	$\mathrm{tf}(t,d)$
movie	16
freedom	7
wallace	43

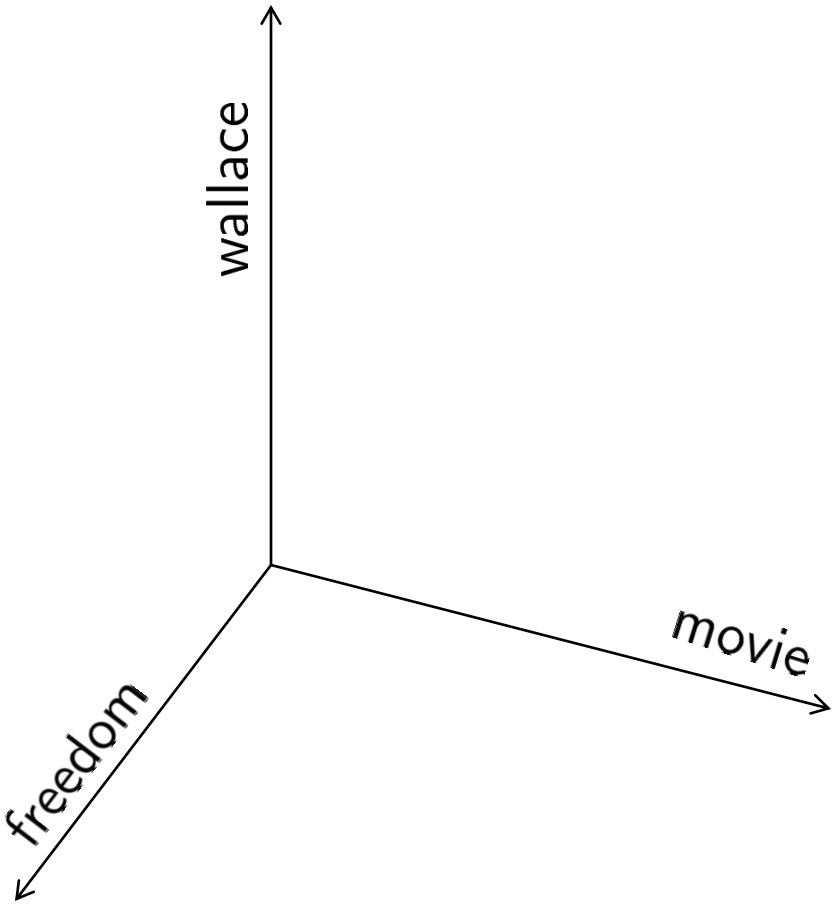
$$l = \sqrt{\sum_{t \in q} \mathrm{tf}(t,d)^2}$$



Vector Space Model (a mention)

t	$\mathrm{tf}(t,d)$	$\mathrm{tf}(t,d)^2$
movie	16	256
freedom	7	49
wallace	43	1,894

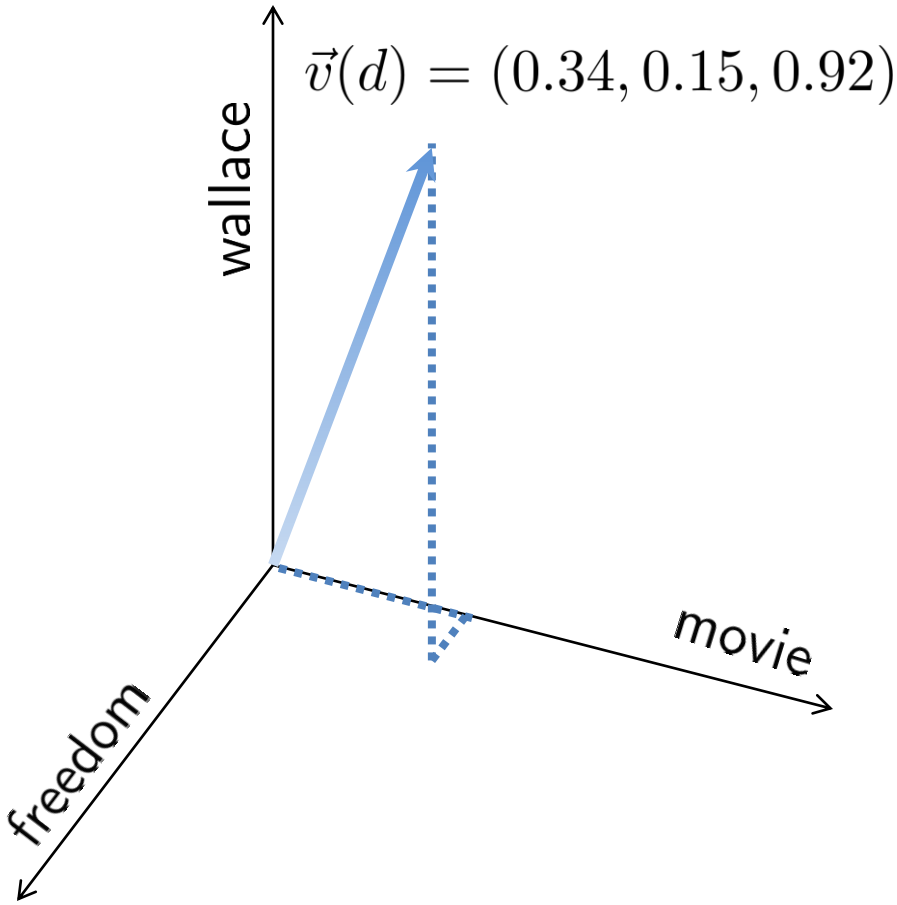
$$l = \sqrt{\sum_{t \in q} \mathrm{tf}(t,d)^2}$$



Vector Space Model (a mention)

t	$\mathrm{tf}(t,d)$	$\mathrm{tf}(t,d)^2$	$\frac{\mathrm{tf}(t,d)}{l}$
movie	16	256	0.34
freedom	7	49	0.15
wallace	43	1,894	0.92

$$l = \sqrt{\sum_{t \in q} \mathrm{tf}(t,d)^2}$$



Dividing by l normalises the length of vector to 1

Vector Space Model (a mention)

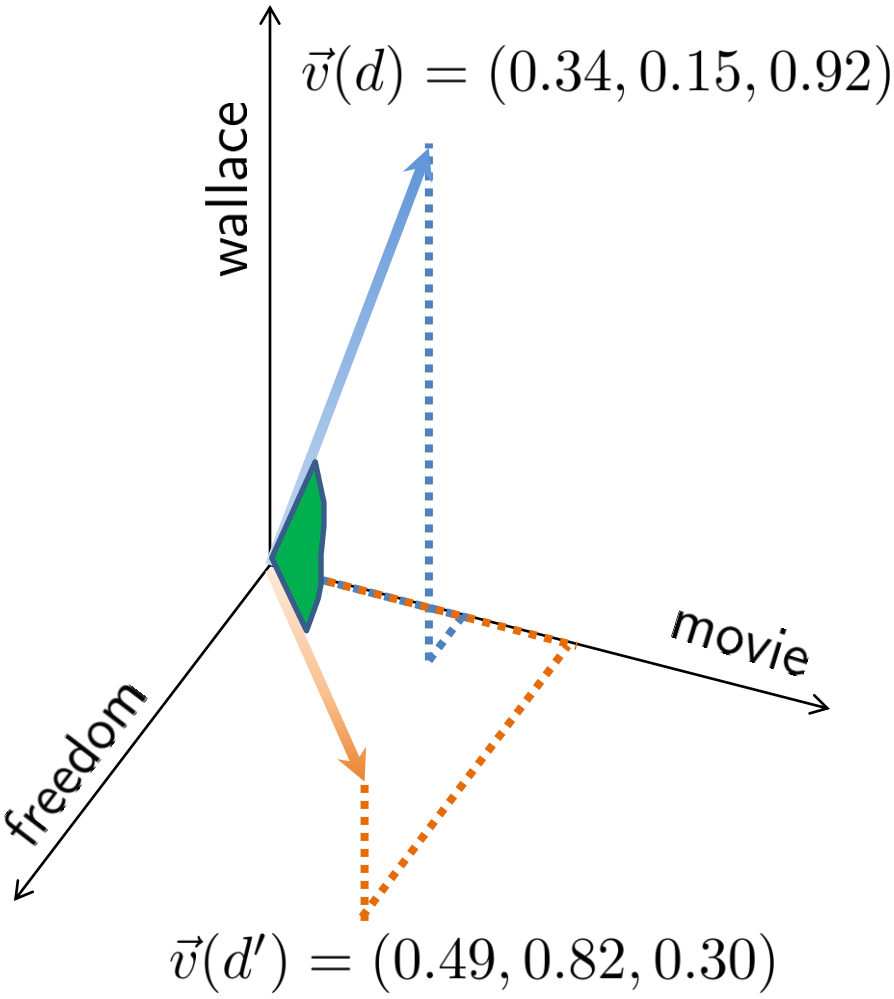
- Cosine Similarity

$$\text{sim}(d, d') = \vec{v}(d) \cdot \vec{v}(d')$$

t	$\vec{v}(d)$	$\vec{v}(d')$	$\times$
movie	0.34	0.49	0.17
freedom	0.15	0.82	0.12
wallace	0.93	0.30	0.28

Σ

$$\text{sim}(d, d') \approx 0.57$$



- Note:

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}|\,|\mathbf{b}| \cos(\angle(\mathbf{a}, \mathbf{b}))$$

$$|\vec{v}(d)| = |\vec{v}(d')| = 1$$

Hence the similarity is the cosine of the **angle** between the vectors

Relevance Measure: TF–IDF

- TF–IDF: Combine Term Frequency and Inverse Document Frequency:

$$\text{tf-idf}(t, d) = \text{tf}(t, d) \times \text{idf}(t, D)$$

- Score for a query
  - Let query $q = (t_1, \ldots, t_n)$
  - Score for a query: $score(q, d) = \sum_{t \in q} \text{tf-idf}(t, d)$

  (There are other possibilities)

… we could also use cosine similarity between query and document using TF–IDF weights

Two Sides to Ranking: Relevance

Field-Based Boosting

- Not all text is equal: titles, headers, etc.

Anchor Text

- See how the Web views/tags a page

```
<!DOCTYPE html PUBLIC "-//W3C//DTD XHTML 1.0 Strict//EN"
 "http://www.w3.org/TR/xhtml1/DTD/xhtml1-strict.dtd">
<html>
<head>
  <title>What I watched last night ...</title>
</head>
<body>
<p>Last night I was pretty bored so I made some popcorn and watched
<a href="http://en.wikipedia.org/Braveheart">a movie about William Wallace called Braveheart</a>.
Set in Scotland it has lots of blood and gore.
</p>
</body>
</html>
```

Anchor Text

- See how the Web views/tags a page

Lucene uses relevance scoring

Tasks Console

SearchWikiIndex [Java Application] C:\Program Files\Java\jre1.8.0_65\bin\javaw.exe (03-05-2017 12:45:22 a. m.)

```
Opening directory at  lucene
Enter a keyword search phrase:
obama
Running query: obama
Parsed query: TITLE:obam^5.0 ABSTRACT:obam
Matching documents: 255
Showing top 10 results
1       http://es.wikipedia.org/wiki/Obama_Republican   Obama Republican
2       http://es.wikipedia.org/wiki/Obama_(Fukui)      Obama (Fukui)
3       http://es.wikipedia.org/wiki/Republicanos_por_Obama     Republicanos por Obama
4       http://es.wikipedia.org/wiki/Engonga_Obame      Engonga Obame
5       http://es.wikipedia.org/wiki/Barack_Obama       Barack Obama
6       http://es.wikipedia.org/wiki/Michelle_Obama     Michelle Obama
7       http://es.wikipedia.org/wiki/Cartel_%22Hope%22_de_Obama Cartel "Hope" de Obama
8       http://es.wikipedia.org/wiki/Transición_presidencial_de_Barack_Obama    Transición presidencial de Barack Obama
9       http://es.wikipedia.org/wiki/Por_qué_Obama_ganará_en_2008_y_en_2012     Por qué Obama ganará en 2008 y en 2012
10      http://es.wikipedia.org/wiki/Ricardo_Mangue_Obama_Nfubea        Ricardo Mangue Obama Nfubea
```

Ranking:
Importance

Two Sides to Ranking: Importance

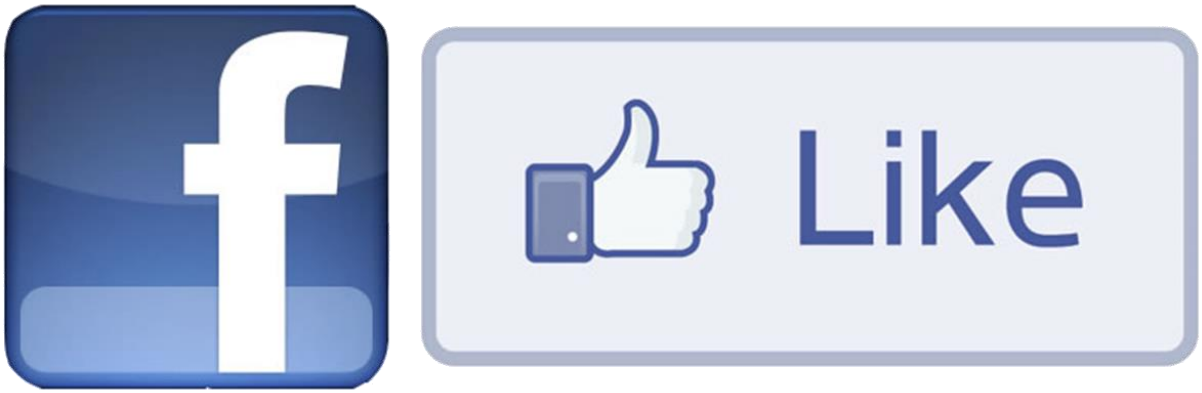
How could we determine that Barack Obama is more important than Mount Obama as a search result for "obama" on the Web?

Link Analysis

Which will have more links from other pages?
The Wikipedia article for Mount Obama?
The Wikipedia article for Barack Obama?

Link Analysis

- Consider links as votes of confidence in a page
- A hyperlink is the open Web's version of …



(… even if the page is linked in a negative way.)

Link Analysis

So if we just count links to a page we can determine its importance and we are done?

Link Spamming

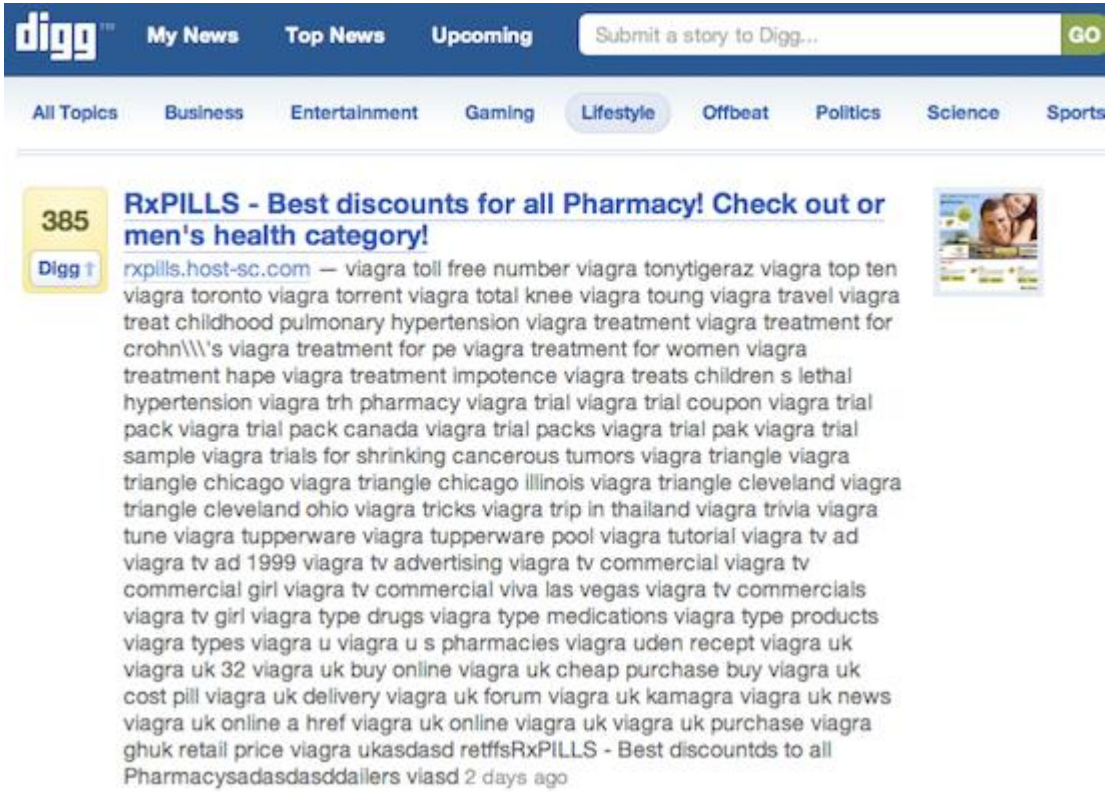
Claritin Clomid Combivent Confido Copegus Cordarone Coreg Coumadin Cozaar Crestor Cyklokapron Cymbalta Cystone Cytotec Danazol Deltasone Depakote Desyrel Detrol Diabecon Diakof Diarex Didronel Differin Dilantin Diovan Dostinex Elavil Elimite Emsam Endep Eurax Evecare Evista Exelon Famvir Feldene Femara Femcare Flomax Flonase Flovent Fosamax Gasex Geodon Geriforte Herbolax High Love Himcocid Himcolin Himcospaz Himplasia Hoodia Hytrin Hyzaar Imdur Imitrex Inderal Ismo Isoptin Isordil Kamagra Karela Keftab Koflet Kytril Lamictal Lamisil Lanoxin Lariam Lasix Lasuna Leukeran Levaquin Levlen Levothroid Lincocin Lioresal Lisinopril Liv.52 Lopid Lopressor Loprox Lotensin Lotrisone Loxitane Lozol Lukol Lynoral Maxaquin Menosan Mentat Mentax Mevacor Mexitil Miacalcin Micardis Mobic Monoket Motrin Myambutol Mycelex-G Mysoline Naprosyn Neurontin Nicotinell Nimotop Nirdosh Nizoral Nolvadex Nonoxinol Noroxin Omnicef Ophthacare Oxytrol Pamelor Parlodel Paxil Penisole Phentrimine Pilex Plan B Plavix Plendil Pletal Prandin Pravachol Prednisone Premarin Prevacid Prilosec Prinivil Procardia Prograf Prometrium Propecia Proscar Protonix Proventil Prozac Purim Purinethol Quibron-T Relafen Renalka Reosto Requip Retin-A Revia Rhinocort Rimonabant Risperdal Rocaltrol Rogaine Rumalaya Sarafem Septilin **Serevent** **Serophene** **Seroquel** Shallaki Shoot Sinequan Singulair Snoroff Sorbitrate Speman Starlix StretchNil Stromectol Styplon Sumycin Superman Sustiva Synthroid Tenormin Topamax Trandate Tricor Trimox Triphala Tulasi Urispas V-Gel Vantin Vasodilan Vasotec Ventolin Viramune Vytorin Xeloda Xenacore Zanaflex Zantac Zebeta Zelnorm Zerit Yerba Diet Wellbutrin SR Women Attracting Pheromones Women's Intimacy Enhancer Women's Intimacy Enhancer Cream Virility Gum Vitamin A & D Viagra + Cialis Viagra + Cialis + Levitra Viagra Jelly Viagra Soft + Cialis Soft Viagra Soft Tabs Ultimate Male Enhancer Toprol XL Touch-Up Kit Tentex Royal Tentex Forte Tiberius Erectus Zero Nicotine 2 Complete Professional Whitening Kits 2 Sets Of Moldable Mouth Trays 36 Beauty Acne-n-Pimple Cream ActoPlus Met Superloss Multi SleepWell (Herbal XANAX) Shuddha Guggulu Rythmol SR Rumalaya Forte Pulmicort Inhaler Professional Plasma Tooth Whitening Kit Premium Diet Patch Penis Growth Oil Penis Growth Pack Penis Growth Patch Penis Growth Pills Orgasm Enhancer Norpace CR Mental Booster Men Attracting Pheromones Menopause Gum Male Enhancement Oil Male Enhancement Patch Male Enhancement Pills Male Sexual Tonic InnoPran XL Hoodia Weight Loss Gum Hoodia Weight Loss Patch Human Growth Hormone Agent Glucotrol XL Green Tea Grifulvin V Gyne-Lotrimin Hair Loss Cream Herbal Maxx Herbal Phentermine Flagyl ER Female Sexual Tonic Female Viagra Epivir-HBV Diet Maxx Deluxe Handheld Plasma Whitening Tool Deluxe Whitening System With Plasma Lamp Coral Calcium Cialis Jelly Cialis Soft Tabs Calcium Carbonate Bust Enhancer Beconase AQ Anatrim Diet Pills Advair Diskus Advanced Gain Pro Breast Augmentation Breast Enhancement Breast Enhancement Gel Breast Enhancement Gum Breast Intense Buy Trazodone Buy Celebrex Buy Alprazolam Buy Tramadol Buy Fioricet Buy Soma Buy Cialis Buy Carisoprodol Buy Levitra Buy Ultram Buy Ambien Buy Viagra Buy Xanax Buy Phentermine Buy Valium Buy Diazepam Generic Celebrex Generic Alprazolam Generic Tramadol Generic Fioricet Generic Soma Generic Cialis Generic Carisoprodol Generic Levitra Generic Ultram Generic Ambien

Link Importance

So which should count for more?
A link from http://en.wikipedia.org/wiki/Barack_Obama?
Or a link from http://freev1agra.com/shop.html?

Link Importance

Maybe we could consider links from some domains as having more “vote”?



PageRank

PageRank

- Not just a count of inlinks
  - A link from a more important page is more important
  - A link from a page with fewer links is more important

  ∴ A page with lots of inlinks from important pages (which have few outlinks) is more important

PageRank is Recursive

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    have few outlin

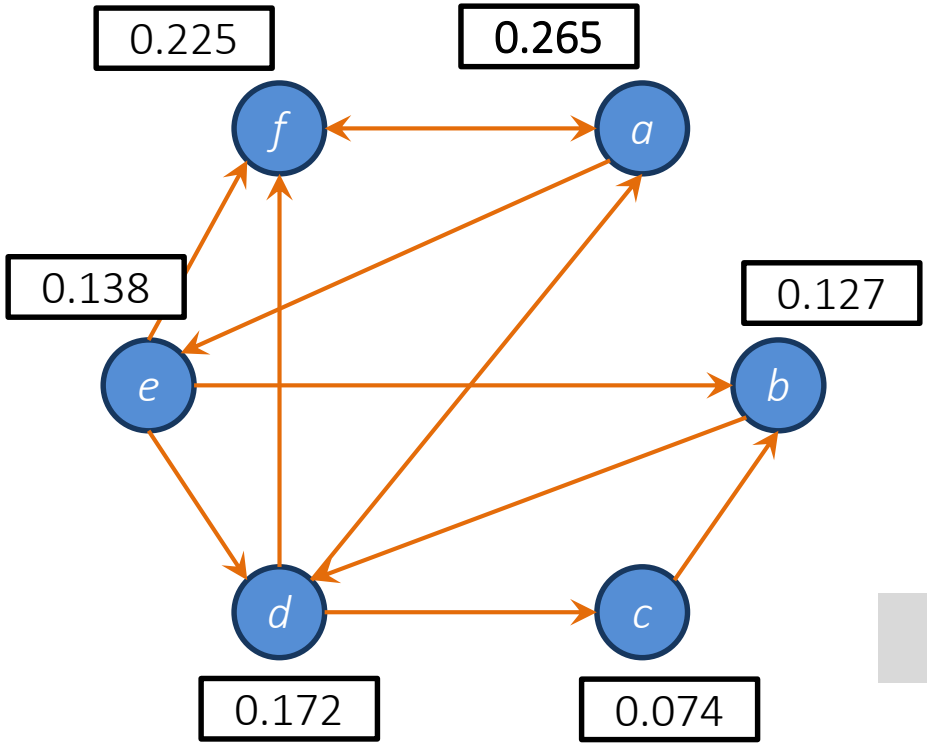
PageRank Model

- The Web: a directed graph

$$G = (V, E)$$

Vertices (*pages*)

Edges (*links*)



Which vertex is most important?

$$V = \{a, b, c, d, e, f\}$$

$$E = \{(a, e), (a, f), (b, d), (c, b), (d, a), (d, c), (d, f), (e, b), (e, d), (e, f), (f, a)\}$$

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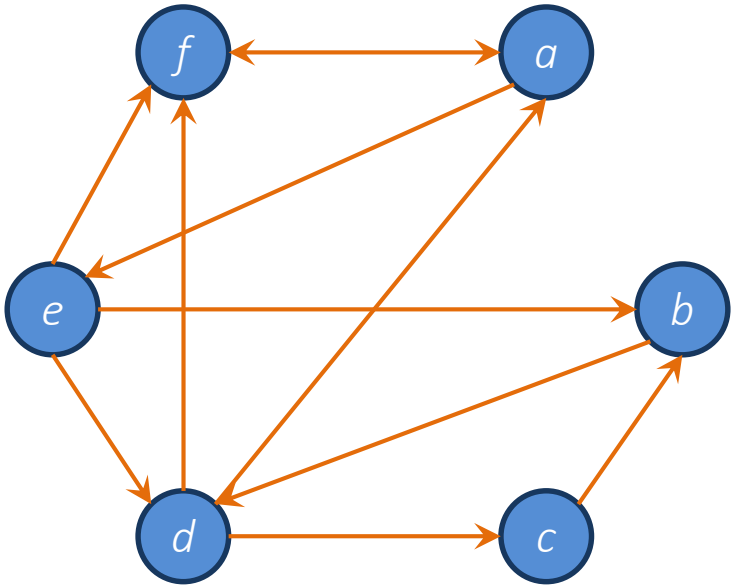

$$\text{out}(v) \coloneqq \{v' \in V \mid (v, v') \in E\}$$

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$$\text{rank}_0(v) \coloneqq \frac{1}{|V|}$$

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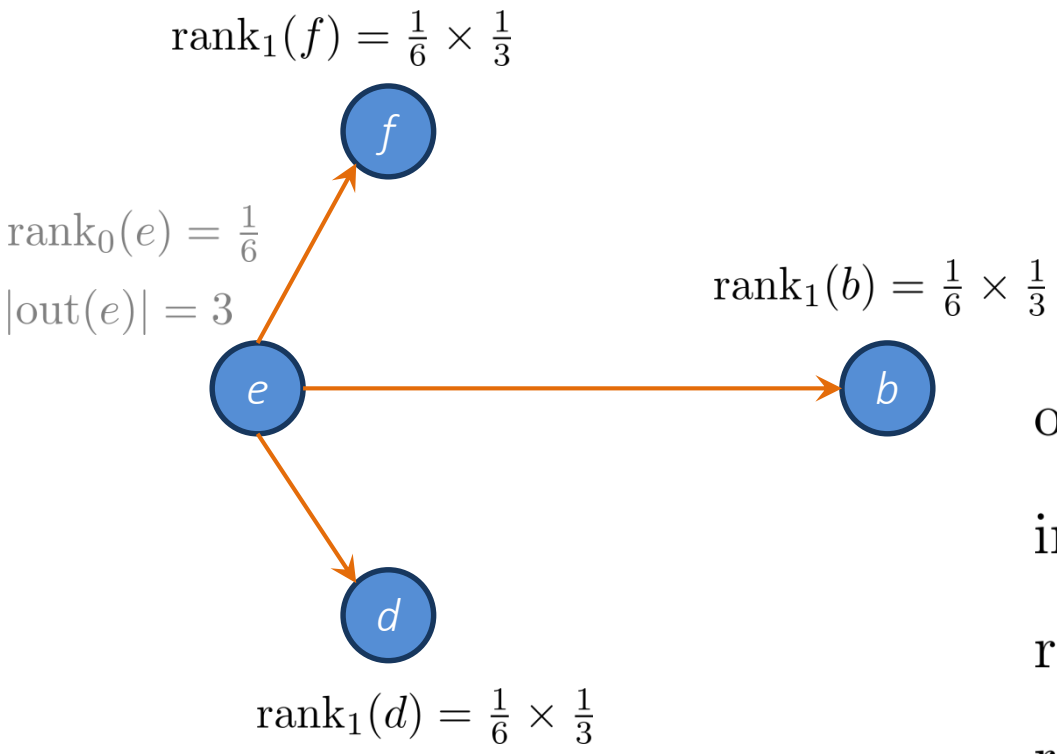

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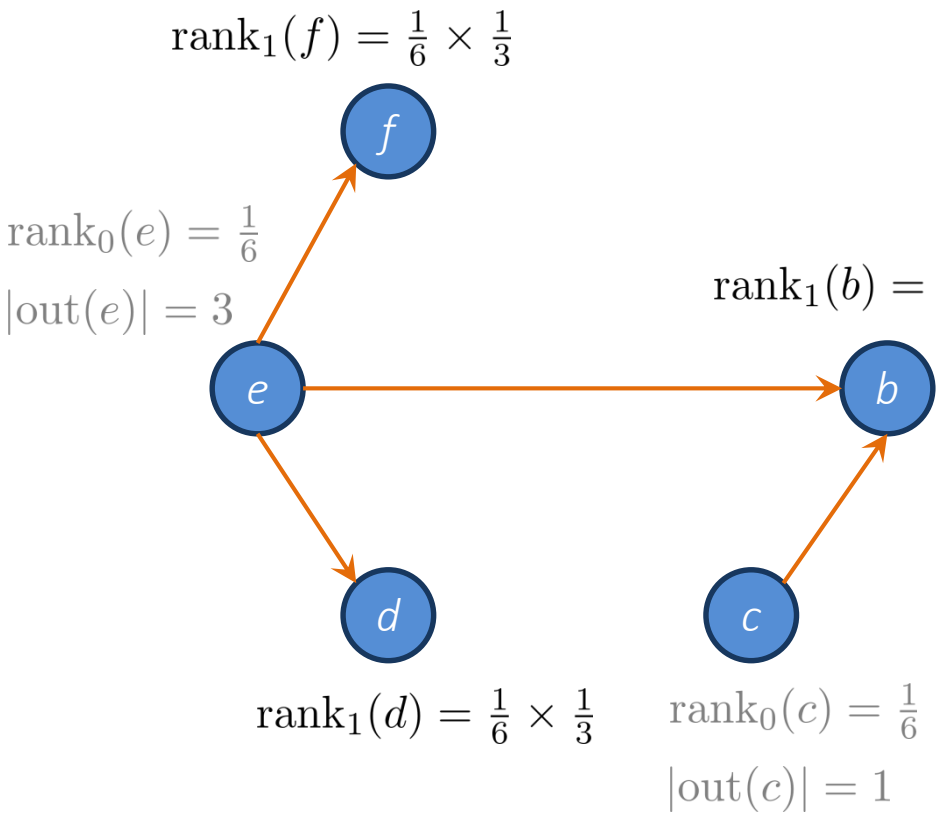

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PageRank: Random Surfer Model

= someone surfing the web, clicking links randomly

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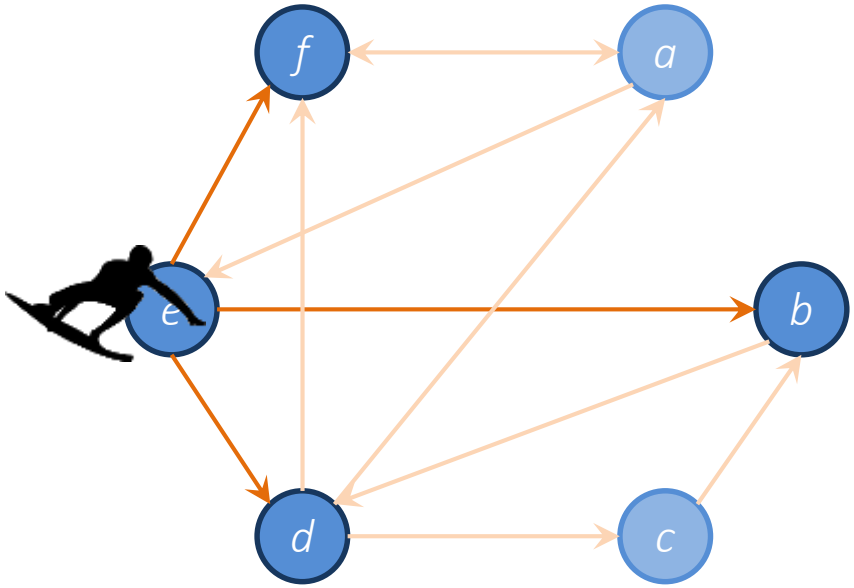

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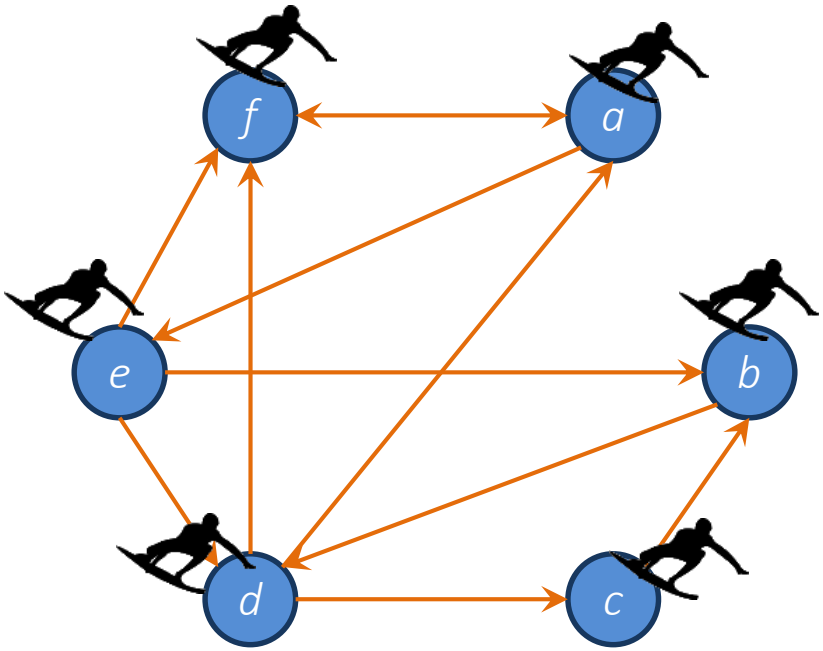

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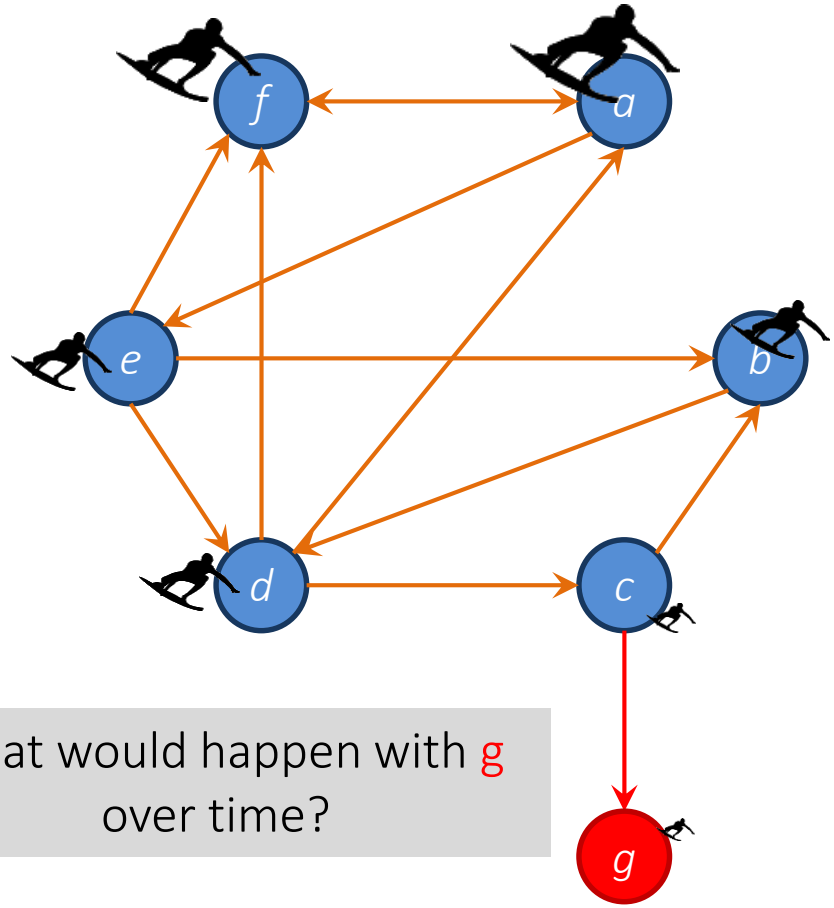

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What would happen with g over time?

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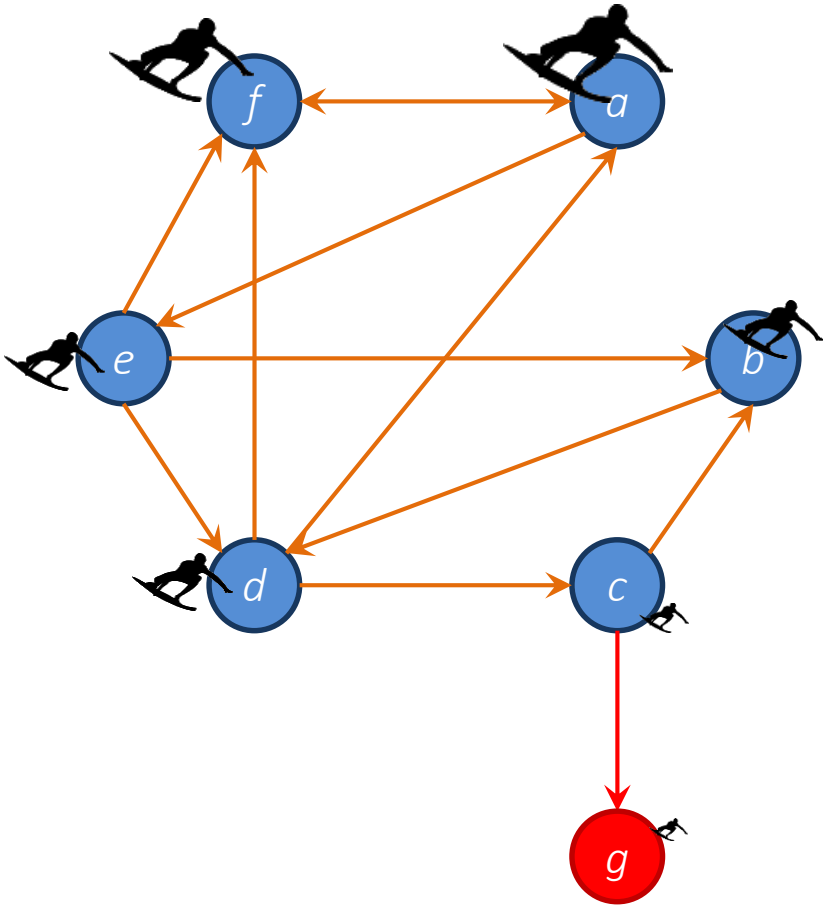

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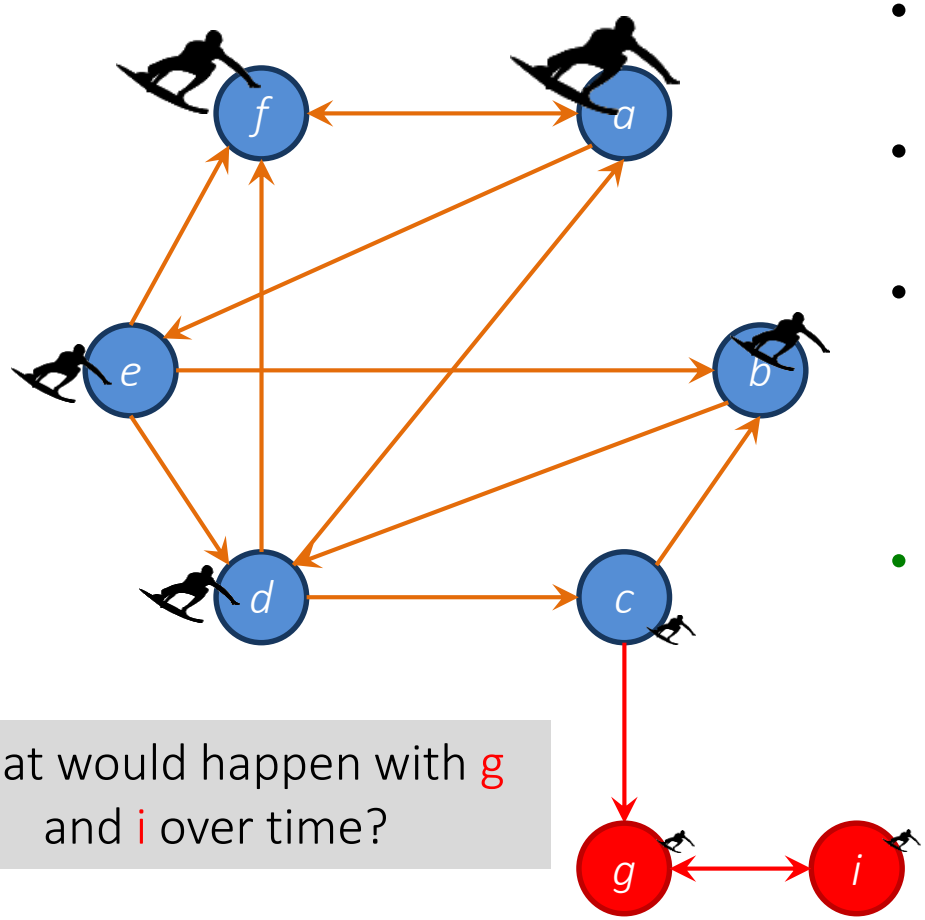

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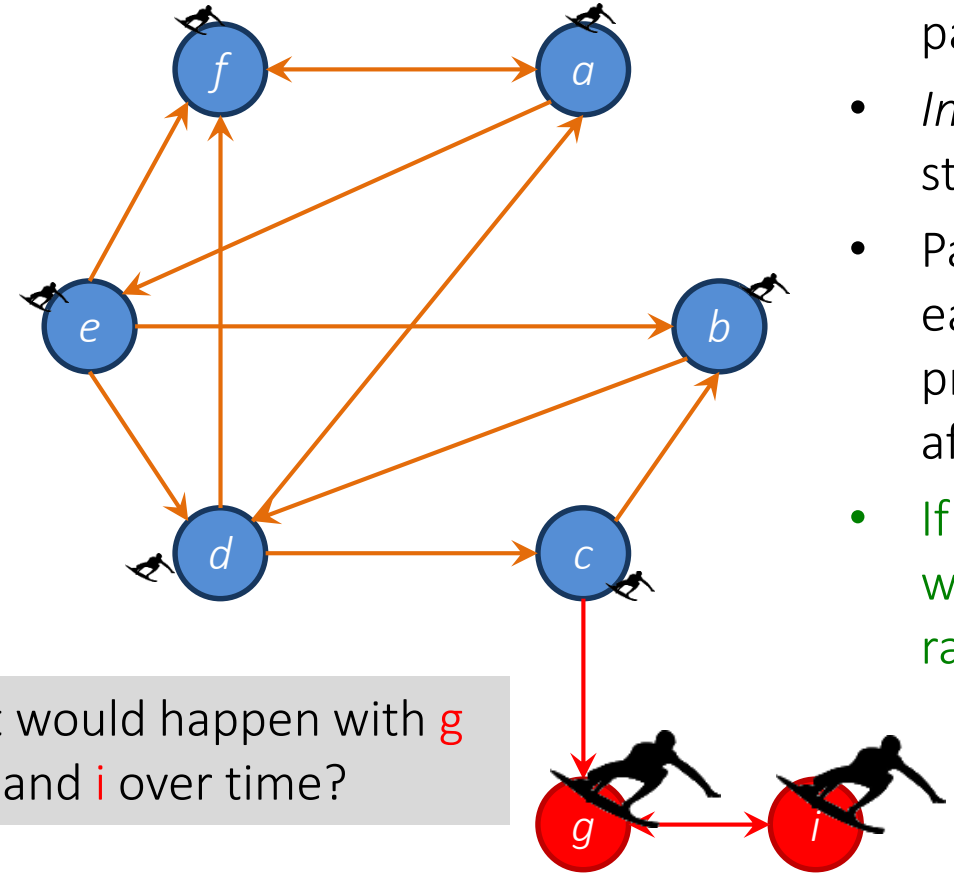

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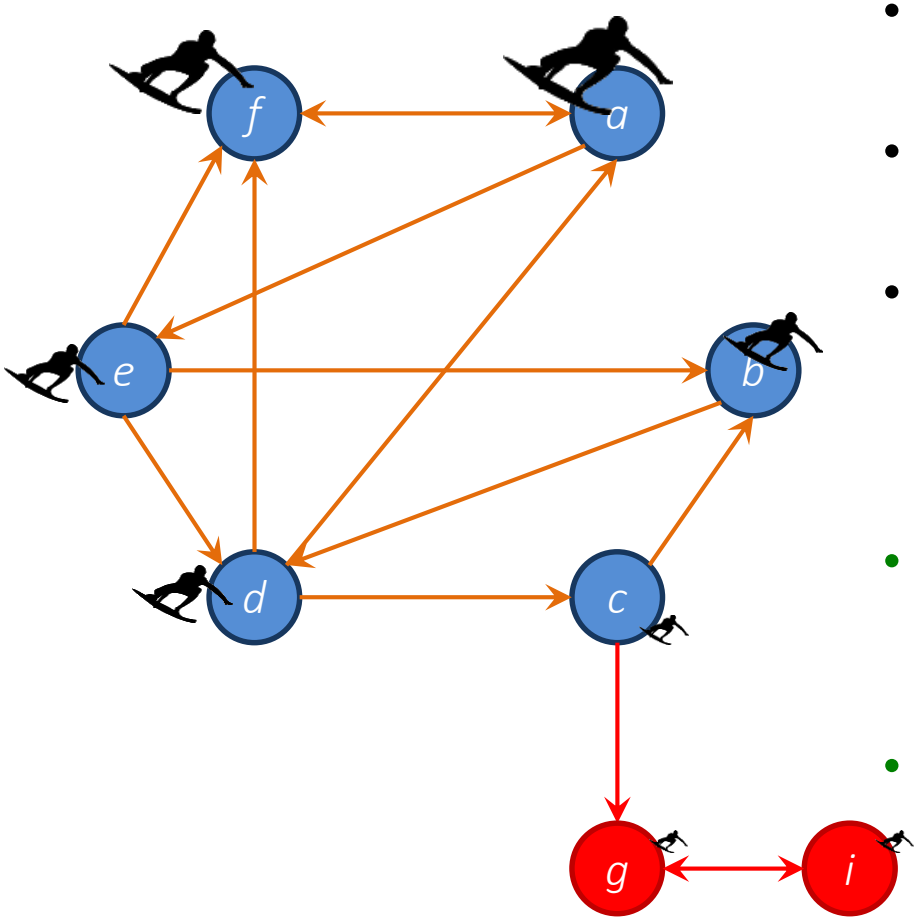

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- If the surfer reaches a page without out-links, the surfer randomly jumps to another page
- The surfer will jump to a random page at any time with a probability 1 – *d … this avoids traps and ensures convergence!*

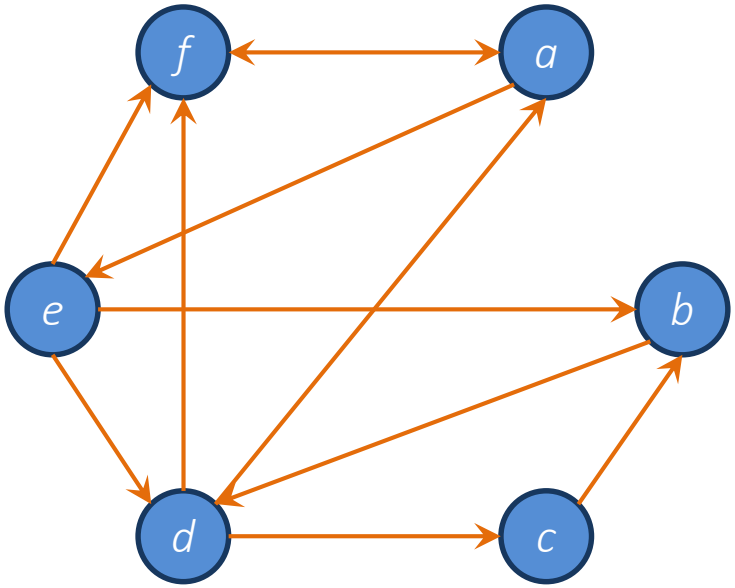
PageRank Model: Final Version

- The Web: a directed graph

$$G = (V, E)$$

Vertices (*pages*)

Edges (*links*)



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$$\text{rank}_0(v) \coloneqq \frac{1}{|V|}$$

$$V' \coloneqq \{v \in V : |\text{out}(v)| = 0\}$$

$$V'' \coloneqq \{v \in V : |\text{out}(v)| \neq 0\}$$

d is the follow-a-link probability

typically $(d = 0.85)$

$$\text{rank}_i(v) \coloneqq d \times \sum_{u \in \text{in}(v)} \frac{\text{rank}_{i-1}(u)}{|\text{out}(u)|} + \sum_{v' \in V'} \frac{\text{rank}_{i-1}(v')}{|V|} + (1-d) \times \sum_{v'' \in V''} \frac{\text{rank}_{i-1}(v'')}{|V|}$$

PageRank: Benefits

- ✓ More robust than a simple link count
- ✓ Fewer ties than link counting
- ✓ Scalable to approximate (for sparse graphs)
- ✓ Convergence guaranteed

Two Sides to Ranking: Importance

Google

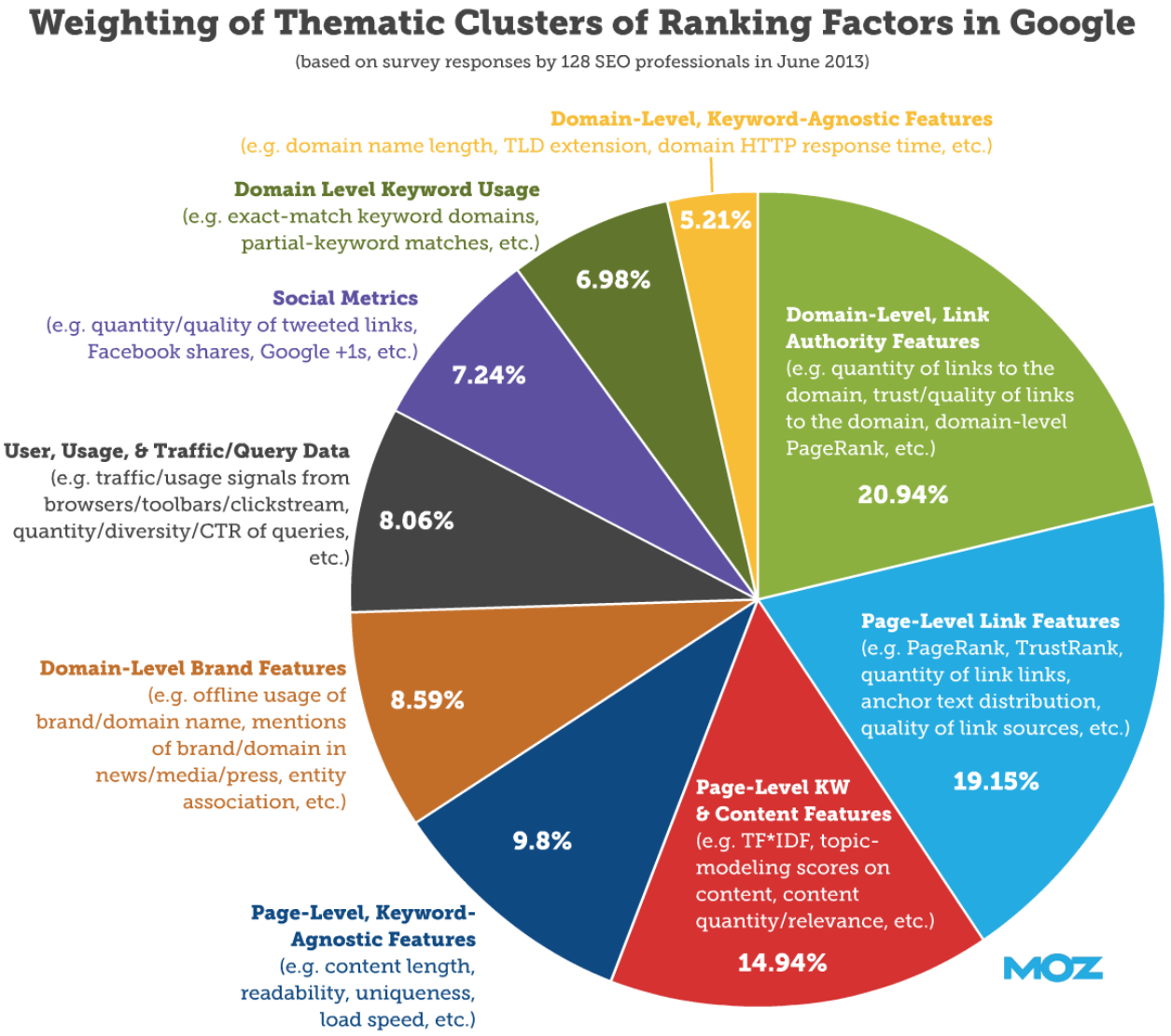
PageRank

>

How does google really rank?

An educated guess

How Modern Google ranks results (maybe)



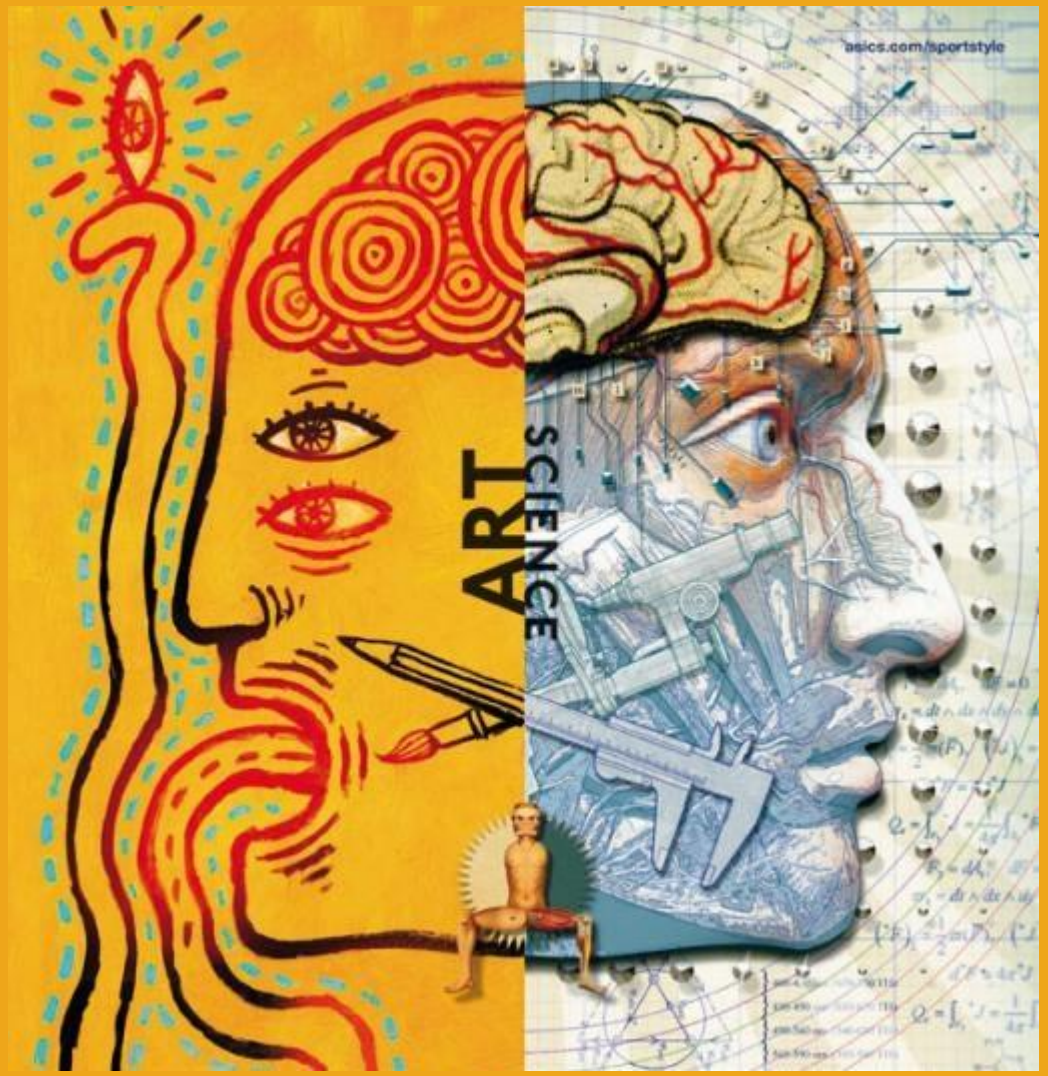
According to survey of SEO experts, not people in Google

How Modern Google ranks results (maybe)

Why so secretive?

According to survey of SEO experts, not people in Google

Ranking: Science or Art?



Questions?